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Rate curves for forward Euribor estimation and CSA-discounting

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Goals

- To provide key elements for rate curve estimation understanding
  - Curve parameterization: discretization and interpolation schemes
  - Bootstrapping algorithm
  - Financial instrument selection

- What is changed since summer 2007
  - How to build multiple forwarding curve
  - Which curve has to be used for discounting

Sections

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1. Rate curve parameterization and interpolation

Rate curve parameterization

- Discrete time-grid of
  - discount factors
  - continuous (sometime compounded) zero rates
  - instantaneous continuous forward rates

$$D(t_i) = \exp(-z(t_i)t_i) = \exp(-\int_0^{t_i} f(t)d\tau)$$

- Only discount factors are well defined at t=0

Interpolation

- Whatever parameterization has been chosen an interpolation for off-grid dates/times is needed
- Discount factors have exponential decay so it makes sense to interpolate on log-discounts
- A (poor) common choice is to interpolate (linearly) on zero rates
- The smoothness of a rate curve is to be measured on the smoothness of its (simple) forward rates. So it would make sense to use a smooth interpolation on (instantaneous continuous) forward rates

The most popular: linear interpolation

- Linear interpolation is
  - Easy
  - Local (it only depends on the 2 surrounding points)

- Linear interpolation on log-discounts generates piecewise flat forward rates

- Linear interpolation on zero rates generates seesaw forward rates

- Linear interpolation on forward rates generates non-smooth forward rates

Smoothness beyond linear: cubic interpolations

- A cubic interpolation is fully defined when the $\{f_i\}$ function values at points $\{x_i\}$ are supplemented with $\{f'_i\}$ function derivative values.

- Different type of first derivative approximations are available:
  - Local schemes (Fourth-order, Parabolic, Fritsch-Butland, Akima, Kruger, etc) use only $\{f_i\}$ values near x_i to calculate each f'_i
  - Non-local schemes (spline with different boundary conditions) use all $\{f_i\}$ values and obtain $\{f'_i\}$ by solving a linear system of equations.

- Local schemes produce C^1 interpolants, while the spline schemes generate C^2 interpolants.

Cubic interpolation problems

- Simple cubic interpolations suffer of well-documented problems such as spurious inflection points, excessive convexity, and lack of locality.
- Wide oscillation can generate negative forward rates.
- Andersen has addressed these issues through the use of shape-preserving splines from the class of generalized tension splines.
- Hagan and West have developed a new scheme based on positive preserving forward interpolation.

Monotonic cubic interpolation: Hyman filter

- Hyman monotonic filter is the simpler, more general, most effective approach to avoid spurious excessive oscillation
- It can be applied to all schemes to ensure that in the regions of local monotoniticity of the input (three successive increasing or decreasing values) the interpolating cubic remains monotonic.

- If the interpolating cubic is already monotonic, the Hyman filter leaves it unchanged preserving all its original features.
- In the case of C^2 interpolants the Hyman filter ensures local monotonicity at the expense of the second derivative of the interpolant which will no longer be continuous in the points where the filter has been applied.

The favourite choice

- Discount factors are a monotonic non-increasing function of *t*: it is reasonable to interpolate on a (log-)discount grid using an interpolation that preserves monotonicity

- My favourite choice is (Hyman) monotonic cubic interpolation on log-discounts
  - Defined in $t=0$
  - Ensure positive rates
  - C^1 on forward rates (C^0 where Hyman filter is really applied)

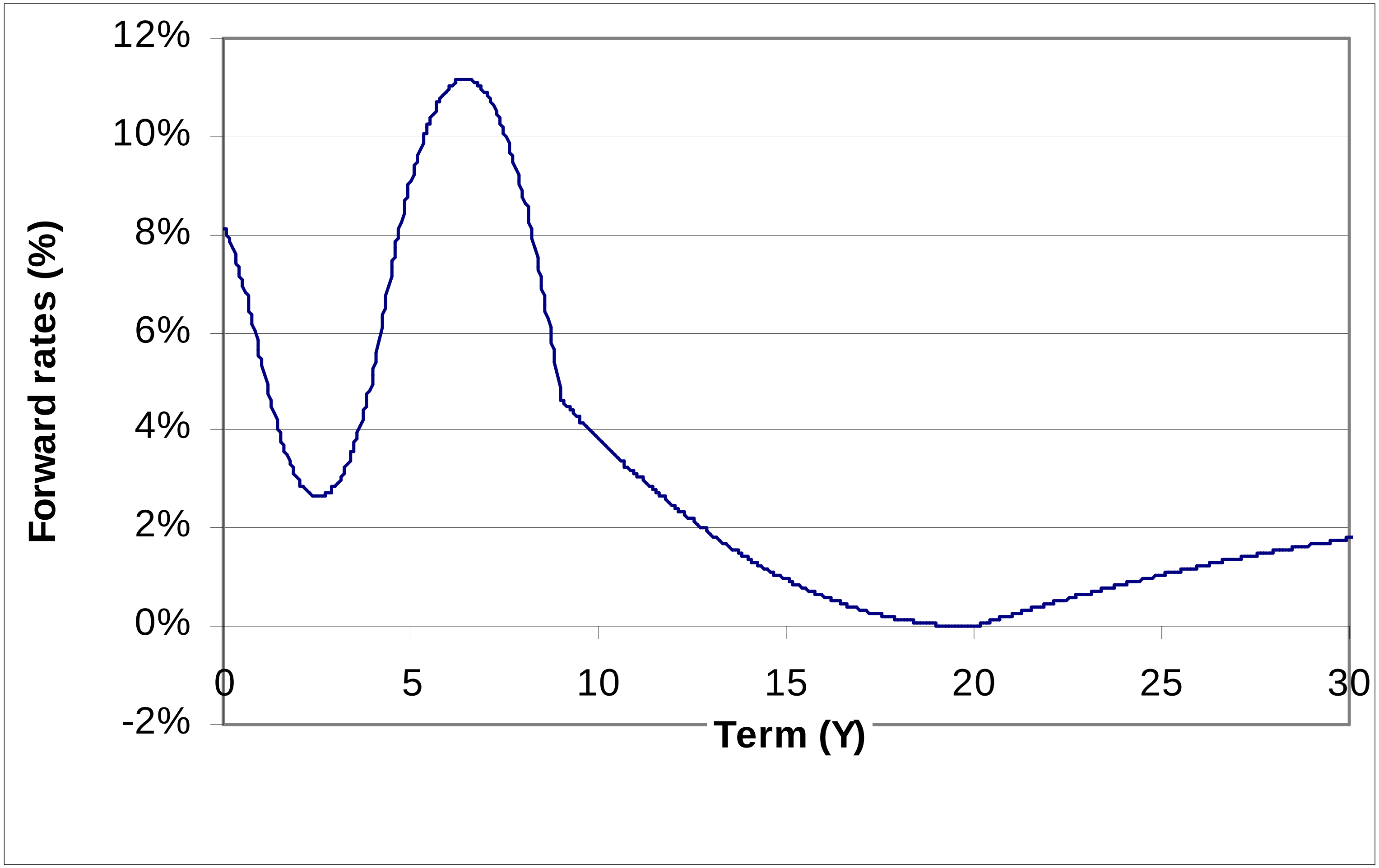
- It's equivalent to (monotonic) parabolic interpolation on forward rates

- Easy to switch to/from linear interpolation on log-discounts to gain robust insight on the curve shape and its problems

Hagan West stress case (1)

Term	Zero rate	Capitalization factor	Discount factor	Log Discount factor	Discrete forward	FRA
0.0	0.00%	1.000000	1.000000	0.000000		
0.1	8.10%	1.008133	0.991933	0.008100	8.1000%	8.1329%
1.0	7.00%	1.072508	0.932394	0.070000	6.8778%	7.0951%
4.0	4.40%	1.192438	0.838618	0.176000	3.5333%	3.7274%
9.0	7.00%	1.877611	0.532592	0.630000	9.0800%	11.4920%
20.0	4.00%	2.225541	0.449329	0.800000	1.5455%	1.6846%
30.0	3.00%	2.459603	0.406570	0.900000	1.0000%	1.0517%

Hagan West stress case (2)



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2. Plain vanilla products

Pillars and financial instruments

- Each time-grid pillar of the rate curve is usually equal to the maturity of a given financial instrument used to define the curve. The so-called interbank curve was usually bootstrapped using a selection from the following market instruments:
  - Deposits covering the window from today up to 1Y;
  - FRAs from 1M up to 2Y;
  - short term interest rate futures contracts from spot/3M (depending on the current calendar date) up to 2Y and more;
  - interest rate Swap contracts from 2Y-3Y up to 30Y, 60Y.

Pillars and financial instruments (2)

- The main characteristics of the above instruments are:
  - they are not homogeneous, having different Euribor indexes as underlying
  - the four blocks overlap by maturity and requires further selection.
- The selection is generally done according to the principle of maximum liquidity:
  - Futures
  - Swaps
  - FRA
  - Deposits

Deposits and FRA

- Interest rate Deposits are OTC zero coupon contracts that start at reference date t_0 (today or spot), span the length corresponding to their maturity, and pay the (annual, simply compounded) interest accrued over the period with a given rate fixed at t_0
  - O/N (*overnight*), T/N (*tomorrow-next*), S/N (*spot-next*)
  - 1W (*spot-week*)
  - 1M, 2M, 3M, 6M, 9M, 12M

- FRAs pay the difference between a given strike and the underlying Euribor fixing.
- 4x7 stands for 3M Euribor fixing in 4 months time
- The EUR market quotes FRA strips with different fixing dates and Euribor tenors.

Euribor futures

- Exchange-traded contracts similar to OTC FRAs. Any profit and loss is regulated through daily marking to market (margining process). Such standard characteristics reduce credit risk and transaction costs, thus enhancing a very high liquidity.

- The most common contracts insist on Euribor3M and expire every March, June, September and December (IMM dates). The first front contract is the most liquid interest rate instrument, with longer expiry contracts having decent liquidity up to about the 8th contract.

- There are also the so called serial futures, expiring in the upcoming months not covered by the quarterly IMM futures. The first serial contract is quite liquid, especially when it expires before the front contract.

- Futures are quoted in terms of prices instead of rates, the relation being rate = 100-price

Convexity adjustment

- Because of their daily marking to market mechanism futures do not have the same payoff of FRAs
  - An investor long a futures contract will have a loss when the futures price increases (rate decreases) but he will finance such loss at lower rate;
  - vice versa when the futures price decreases the profit will be reinvested at higher rate.

- Forward rate volatility and its correlation to the spot rate have to be accounted for.
- Easiest evaluation using Hull-White (Bloomberg: fixed mean reversion, rough volatility evaluation)

- A convexity adjustment is needed to convert the rate implied in the futures price to its corresponding FRA rate: 100-Fut = FRA - Conv

Interest Rate Swaps

- Swaps are OTC contracts in which two counterparties agree to exchange fixed against floating rate cash flows.

- The EUR market quotes standard plain vanilla swaps starting at spot date with annual fixed leg versus floating leg indexed to 6M (or 3M) Euribor rate

- Swaps can be regarded as weighted portfolios of 6M (or 3M) FRA contracts

Basis swaps

- Interest rate (single currency) Basis Swaps are usually floating vs floating swaps with different tenors on the two legs

- The EUR market quotes standard plain vanilla basis swaps as portfolios of two regular fixed-floating swaps with the floating legs paying different Euribor indexes.
- The quotation convention is to provide the difference (in basis points) between the fixed rate of the two regular swaps.

- Basis is positive and decreasing with maturity, reflecting the preference of market players for receiving payments with higher frequency (e.g. 3M instead of 6M, 6M instead of 12M, etc.) and shorter maturities.
- Basis swaps allow to imply levels for non-quoted swaps on Euribor 1M, 3M, and 12M from the quoted swap rates on Euribor 6M

Overnight indexed swaps

- Fixed interest rate is exchanged for the overnight rate.
- The overnight rate is compounded and paid at maturity.
- On both legs there is a single payment for maturity up to 1Y, yearly payments with short stub for longer maturities

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3. Rate Curve Bootstrapping

Current rate curve

- Pricing complex interest rate derivatives requires modeling the *future* dynamics of the rate curve term structure. But any modeling approach will fail to produce good/reasonable prices if the *current* term structure is not correct.
- Most of the literature assumes the existence of the current rate curve as given and its construction is often neglected, or even obscured.
- Financial institutions, software houses and practitioners have developed their own proprietary methodologies in order to extract the rate curve term structure from quoted prices of a finite number of liquid market instruments.
- It is more an art than a science

Best or exact fit

- Best-fit algorithms assume a smooth functional form for the term structure and calibrate their parameters by minimizing the re-pricing error of a chosen set of calibration instruments.
  - Popular due to the smoothness of the curve, calibration easiness, intuitive financial interpretation of functional form parameters (often level, slope, and curvature in correspondence with the first three principal components).
  - The fit quality is typically not good enough for trading purposes in liquid markets.

- Exact-fit algorithms are often preferred: they fix the rate curve on a time grid of *N* pillars in order to exactly re-price *N* calibration instruments.

Bootstrapping

- The *bootstrapping* algorithms is (often) incremental, extending the rate curve step-by-step with the increasing maturity of the ordered instruments
- Intermediate rate curve values are obtained by interpolation on the bootstrapping grid.
- Little attention has been devoted in the literature to the fact that interpolation is often already used during bootstrapping, not just after that, and that the interaction between bootstrapping and interpolation can be subtle if not nasty

Bootstrapping and interpolation

- When using non-local interpolation the shape of the already bootstrapped part of the curve is altered by the addition of further pillars.

- This is usually remedied by cycling in iterative fashion: after a first bootstrap the resulting complete grid is altered one pillar at time using again the same bootstrapping algorithm, until convergence is reached.

- The first iteration can use a local interpolation scheme to reach a robust first guess
- Even better: use a good grid guess, the most natural one being just the previous state grid in a dynamically changing environment.

The standard rate curve

- ON, TN (for curve defined from today)
- Spot: SN, SW, 1M, 2M, etc. (at least up to the first IMM date)
- Futures (8 contracts, maybe one serial)
- Swaps (2Y, 3Y, .., 30Y and beyond)

Some warnings

- Naive algorithms may fail to deal with market subtleties such as
  - date conventions
  - the intra-day fixing of the first floating payment of a swap
  - the futures convexity adjustment
  - the turn-of-year effect

- Note that all instruments are calibrated zeroing their NPV on the boostrapped curve. This is equivalent to zeroing their only cashflow for all instruments but swaps.
- Swaps NPV zeroing depends on the discount curve.

QuantLib Approach: interpolated curves

```
template <class Interpolator>
class InterpolatedDiscountCurve

template <class Interpolator>
class InterpolatedZeroCurve

template <class Interpolator>
class InterpolatedForwardCurve


template <class Traits, class Interpolator,
       template <class> class Bootstrap = IterativeBootstrap>
class PiecewiseYieldCurve
```

QL Approach: bootstrapping instrument wrappers

```
template <class TS>
class BootstrapHelper : public Observer , public Observable {
   public :
     BootstrapHelper(const Handle<Quote>& quote);
     virtual ~BootstrapHelper() {}
     Real quoteError() const;
     const Handle<Quote>& quote() const;
     virtual Real impliedQuote() const = 0;
     virtual void setTermStructure(TS*);
     virtual Date latestDate() const;
     virtual void update();
   protected :
     Handle<Quote> quote_ ;
     TS* termStructure_ ;
     Date latestDate_ ;
};
```

QuantLib Approach: iterative bootstrap (1)

```
template <class Curve>
void IterativeBootstrap<Curve>::calculate() const {

    Size n = ts_->instruments_.size();
    // sort rate helpers by maturity
    // check that no two instruments have the same maturity
    // check that no instrument has an invalid quote
    for (Size i=0; i<n ; ++i)
      ts_->instruments_[i]->setTermStructure(const_cast<Curve*>(ts_));
    ts_->dates_ = std::vector<Date>(n+1);
    // same for the time & data vectors
    ts_->dates_[0] = Traits::initialDate(ts_);
    ts_->times_[0] = ts_->timeFromReference (ts_->dates_[0]);
    ts_->data_[0] = Traits::initialValue(ts_);
    for (Size i=0; i<n; ++i) {
      ts_->dates_[i+1] = ts_->instruments_[i]->latestDate();
      ts_->times_[i+1] = ts_->timeFromReference(ts_->dates_[i+1]);
    }
```

QuantLib Approach: iterative bootstrap (2)

```
  Brent solver;
  for (Size iteration=0;  ; ++iteration) {
    for (Size i=1; i<n+1; ++i) {
      if (iteration==0) {
        // extend interpolation a point at a time
        ts_->interpolation_=ts_->interpolator_.interpolate(
                      ts_->times_.begin(), ts_->times_.begin()+i+1,
                      ts_->data_.begin());
         ts_->interpolation_.update();
       }
       Rate guess, min, max;
       // estimate guess using previous iteration's values,
       // extrapolating, or asking the traits, then bracket
       // the solution with min and max
       BootstrapError<Curve> error(ts_, instrument, i);
       ts_->data_[i]=solver.solve(error, ts_->accuracy_, guess,min,max);
     }
     if (! Interpolator::global)
       break ; // no need for convergence loop
     // check convergence and break if tolerance is reached, bail out
     // if tolerance wasn't reached in the given number of iterations
   }
}
```

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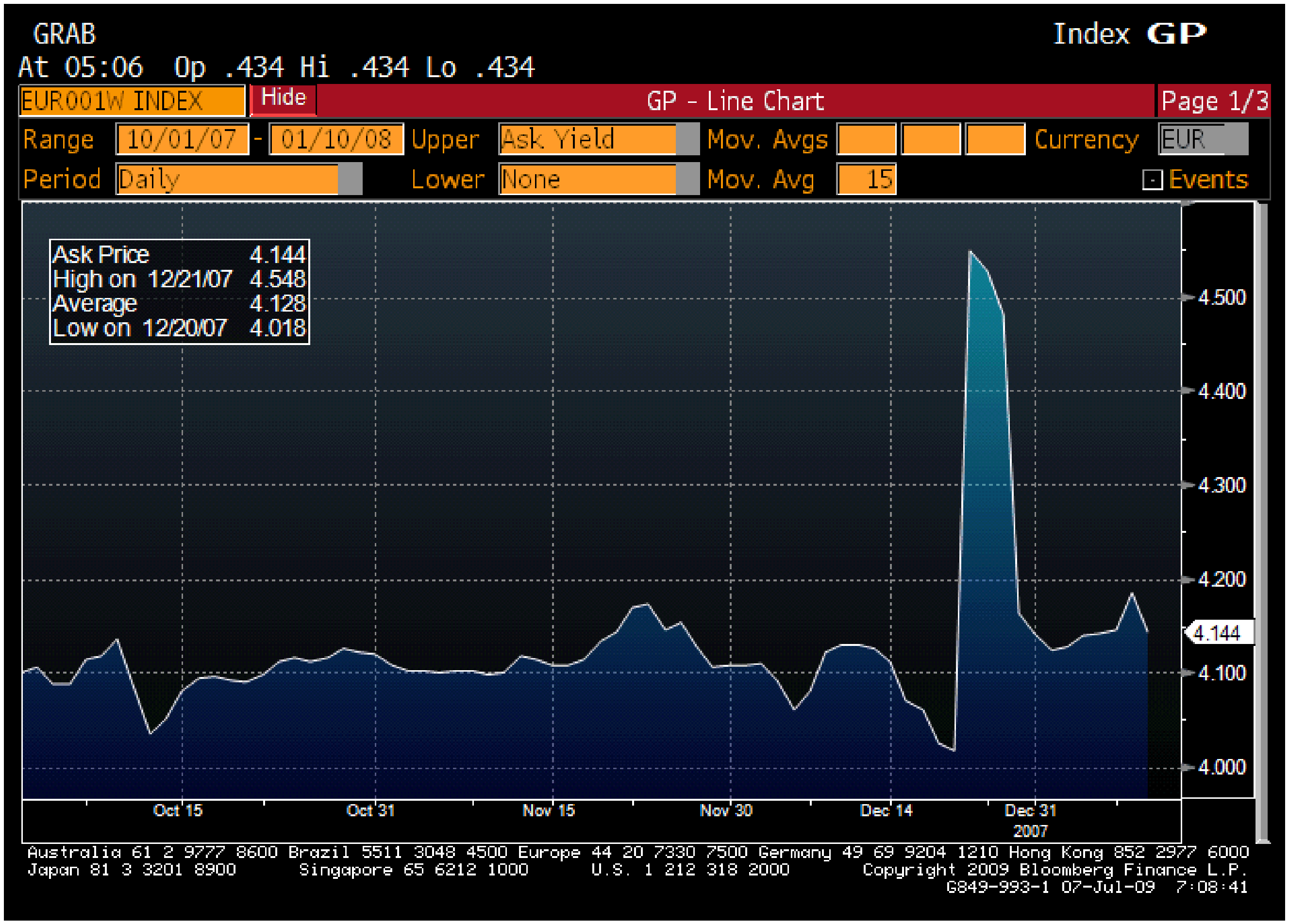
4. Turn Of Year

Smoothness and jumps

- Smooth forward rates is the key point of state-of-the-art bootstrapping.
- For even the best interpolation schemes to be effective any market rate jump must be removed, and added back only at the end of the smooth curve construction.
- The most relevant jump in rates is the so-called turn of year effect, observed in market quotations of rates spanning across the end of a year.
- From a financial point of view, the TOY effect is due to the increased search for liquidity by year end for official balance sheet numbers and regulatory requirements.

Turn of year (TOY) effect



Jump amplitude

- The larger jump is observed the last working day of the year (e.g. 31th December) for the Overnight Rate

- Other Euribor indexes with longer tenors display smaller jumps when their maturity crosses the same border:
  - the Euribor 1M jumps 2 business days before the 1st business day of December;
  - the Euribor 3M jumps 2 business days before the 1st business day of October;
  - Etc.

- There is a decreasing jump amplitude with increasing rate tenor. Think of 1M Euribor as an average of 22 (business days in one month) overnight rates (plus a basis). If this 1M Euribor spans over the end of year, the TOY overnight rate weights just 1/22th. For rates with longer tenors the TOY overnight rate has even smaller weight.

How many TOYs ?

- The December IMM futures always include a jump, as well as the October and November serial futures
- 2Y Swaps always include two jumps; etc.

- The effect is generally observable at the first two TOYs and becomes negligible at the following ones.

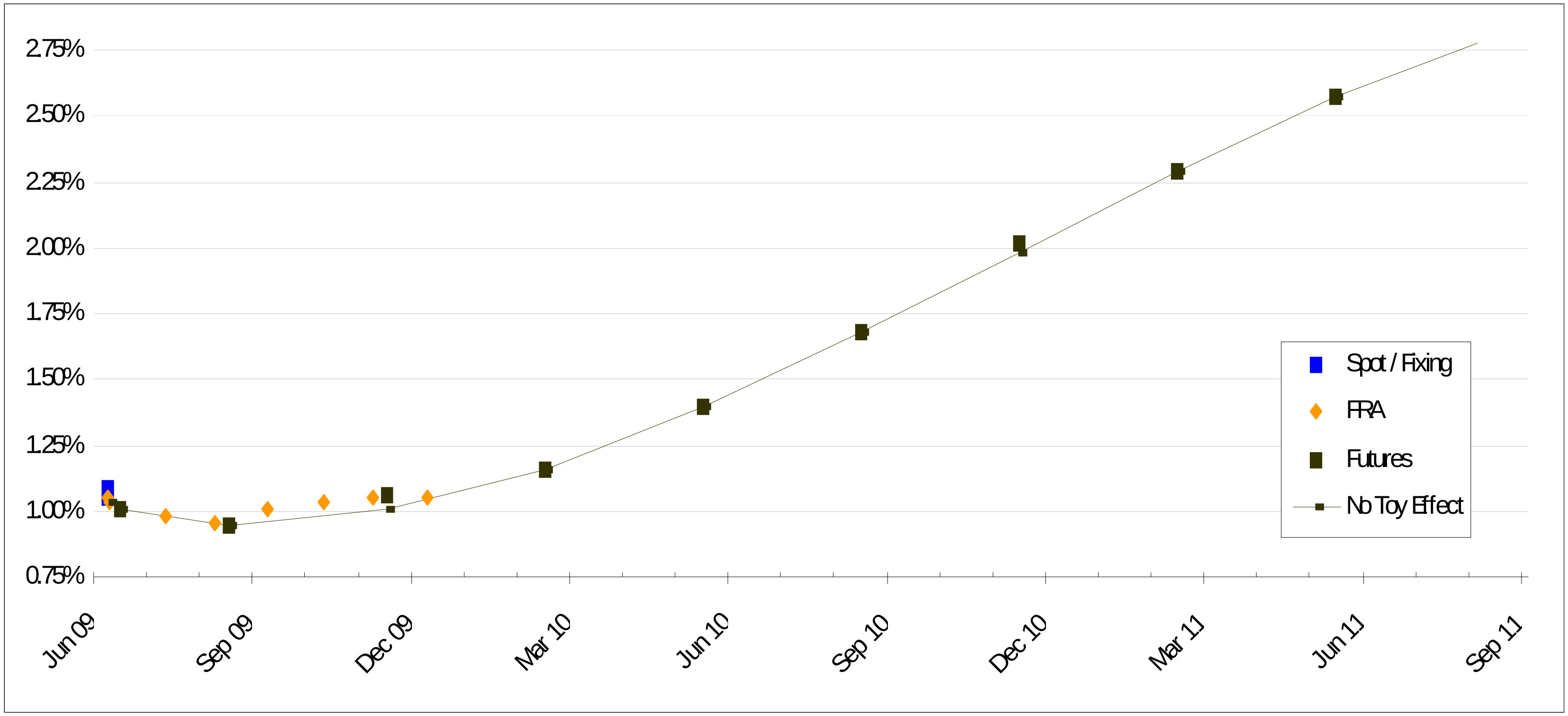
TOY estimation using 3M futures strip

- a fictitious *non-jumping* December rate is obtained through interpolation of surrounding non-TOY non-jumping rates;
- the jump amplitude is the difference between this fictitious December rate and the real one

- Given eight liquid futures this approach always allows the estimation of the second TOY.
- The first TOY can be estimated only up to (two business days before) the September contract expiration: later in the year the first TOY would be extrapolated, which is non robust

Euribor 3M: TOY effect

- Strip 3M

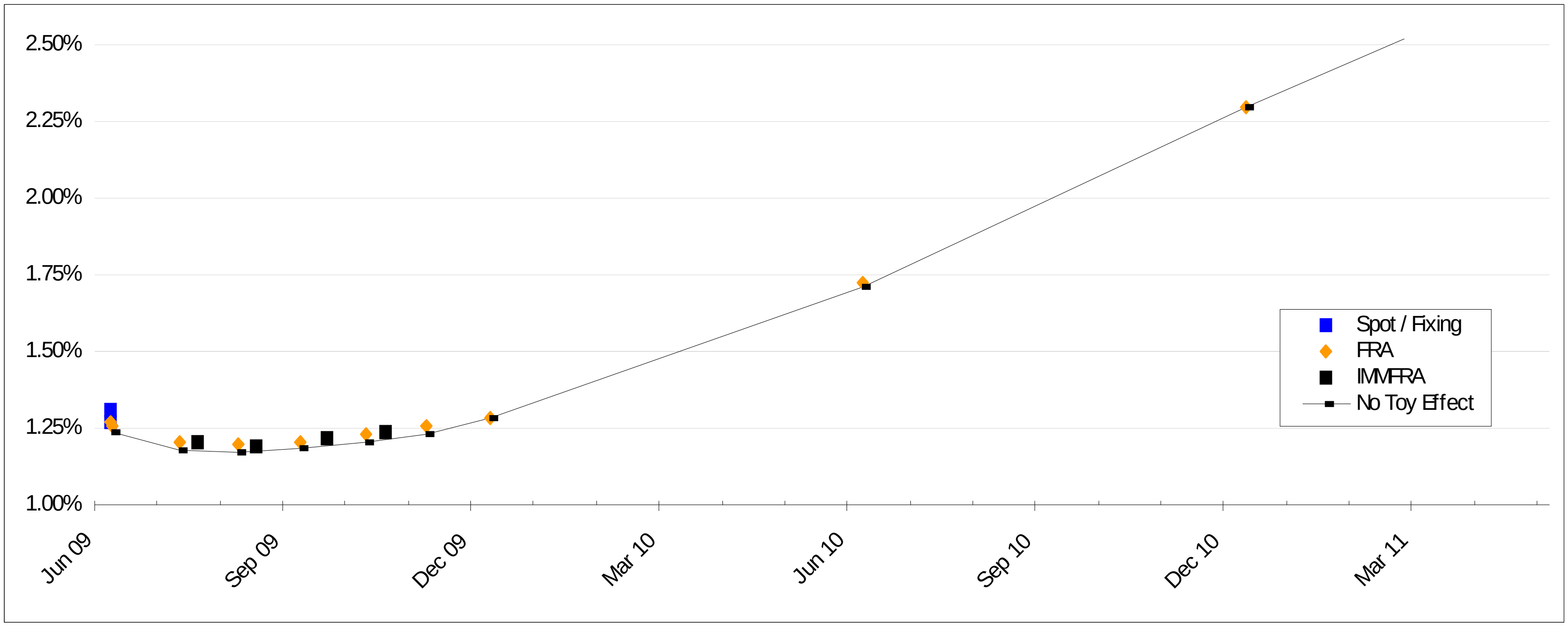


Alternative first TOY estimations

- With the same approach one can use
  - 6M FRA sequence up to (two business days before) the first business day of July
  - 1M swap strip up to (two business days before) the first business day of December
- All these empirical approaches, when available at the same time, give estimates in good agreement with each other.

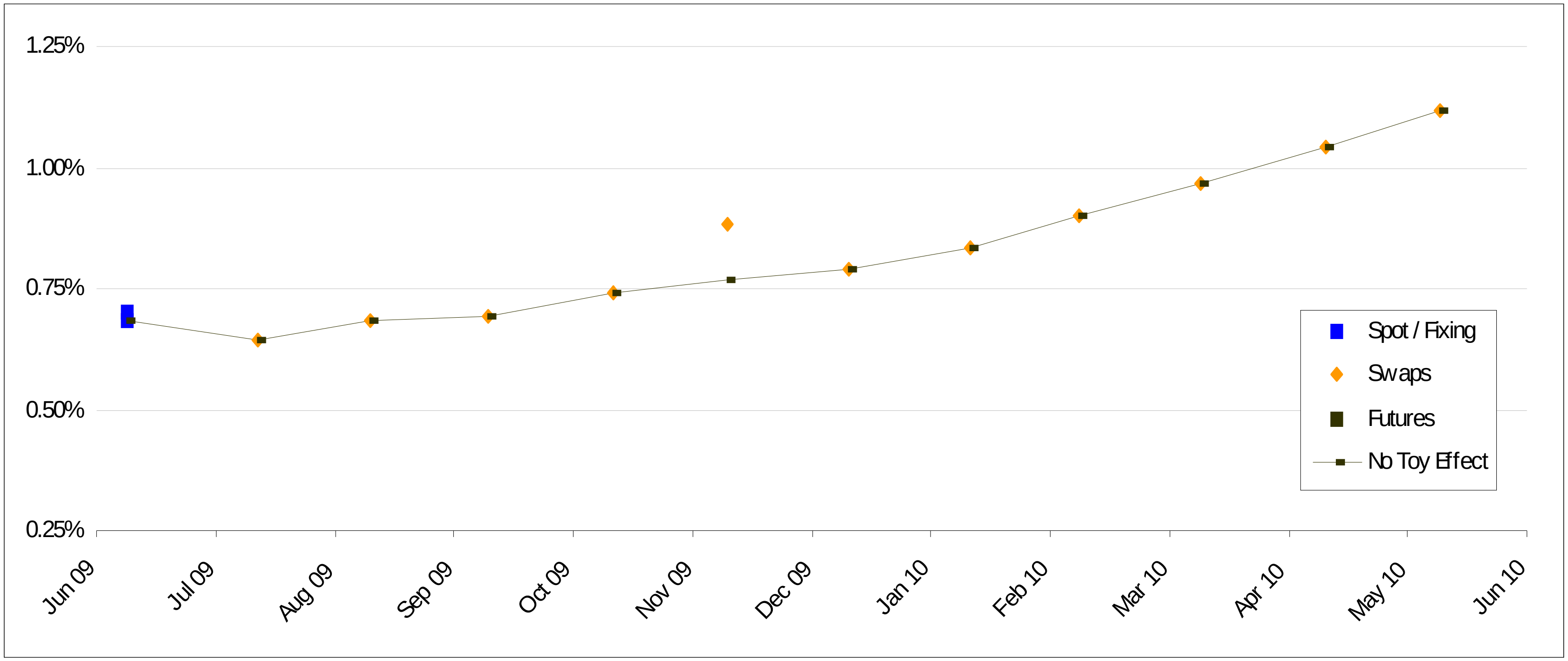
Euribor 6M: TOY effect

- Strip 6M



Euribor 1M: TOY effect

- Strip 1M



RESET

- RESET is a weekly FRA strip consensus average.
- This approach is valid all year long, but it allows only a discontinuous weekly update.

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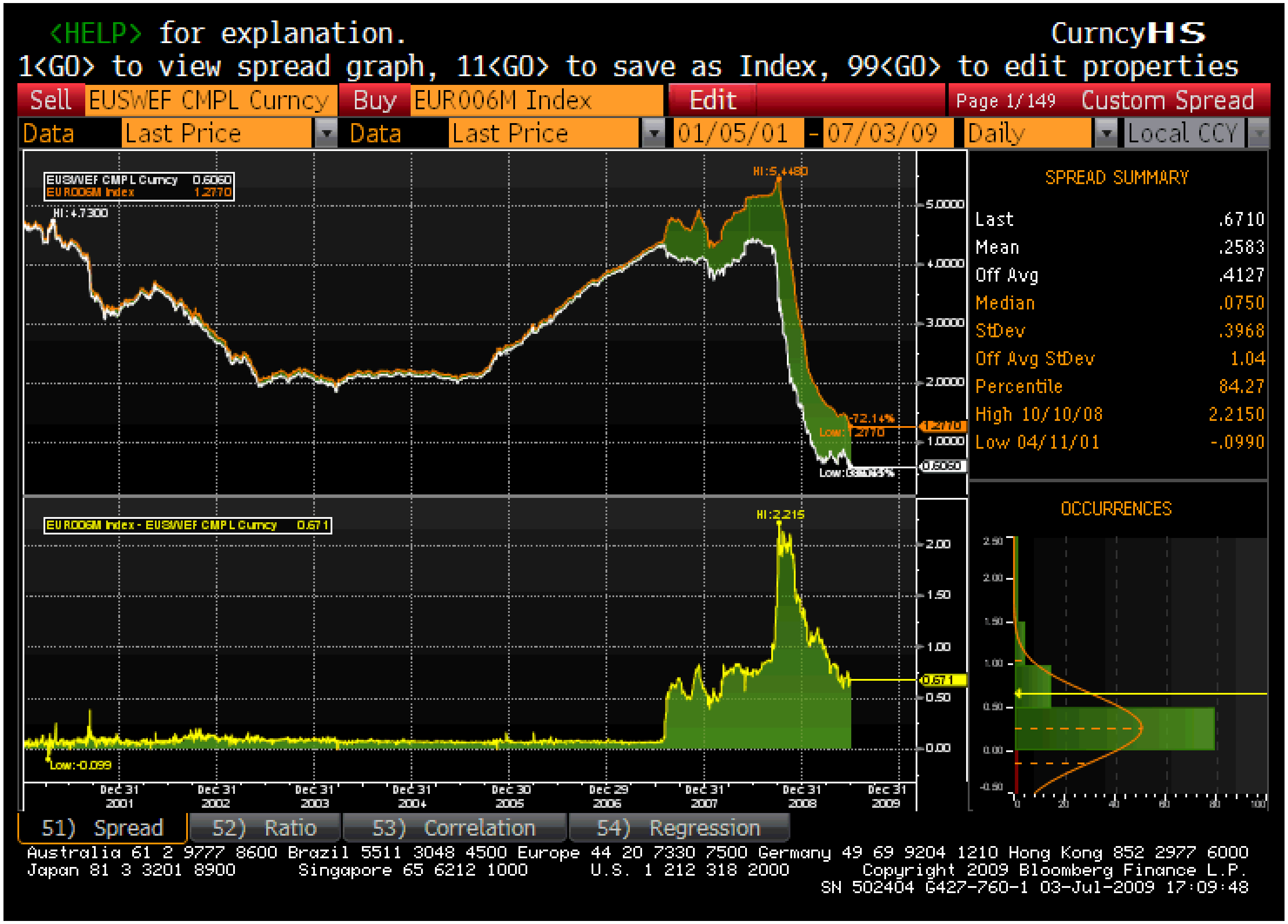
5. What has changed

EURIBOR 6M vs EONIA SWAP 6M



BASIS SWAP 3M vs 6M maturity 5Y

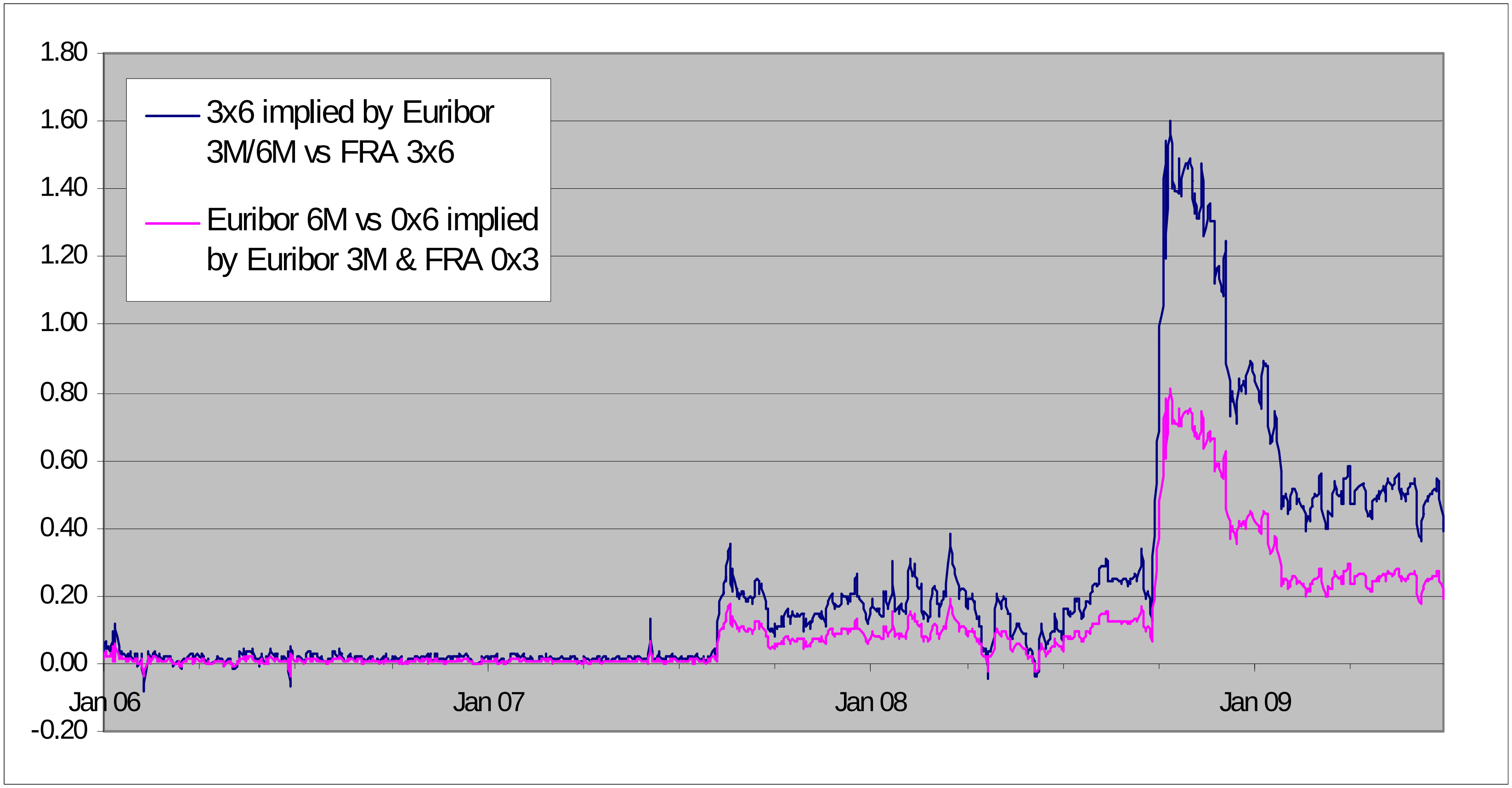
What's New

- Higher basis spreads observed on the interest rate market since summer 2007 reflect increased credit/liquidity risk and the corresponding preference for higher frequency payments (quarterly instead of semi-annually, for instance).

- These large basis spreads imply that different rate curves are required for market coherent estimation of forward rates with different tenors

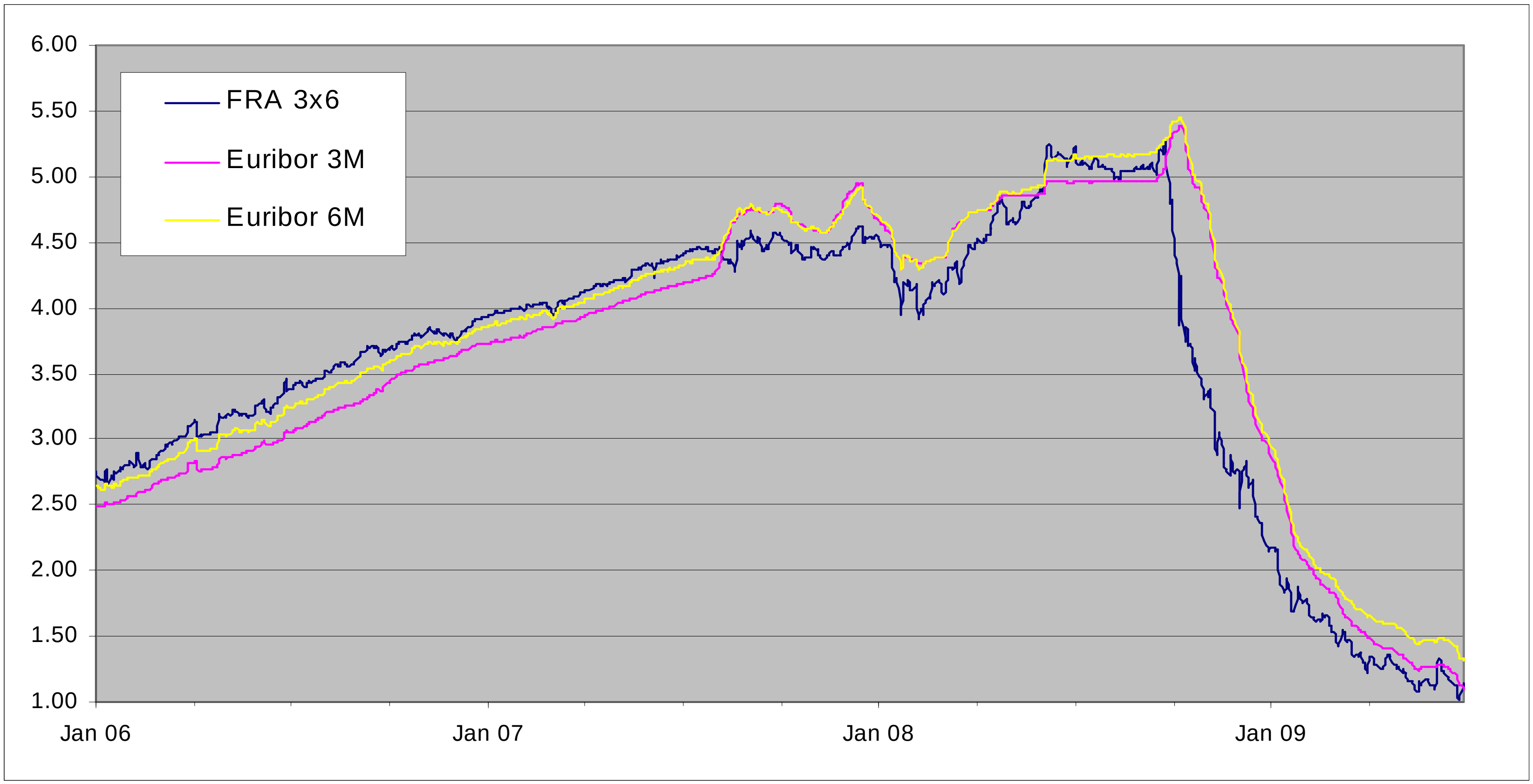
The end of the 3x6 FRA textbook example

{ 3M Euribor, 3x6 } != 6M Euribor



The end of the 3x6 FRA textbook example

It’s not a correlation break



The death of the single rate curve

- Alternative empirical evidences that a single curve cannot be used to estimate forward rates with different tenors:
  - two consecutive futures are not in line with their spanning 6M FRA
  - FRA and futures rates are not in line with EONIA based Overnight Indexed Swaps over the same period

- One single curve is not enough anymore to account for forward rates of different tenor, such as 1M, 3M, 6M, 12M

- Even sophisticated old-school bootstrapping algorithms fail to estimate correct forward Euribor rates in the new market conditions observed since the summer of 2007

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6. Forwarding Rate Curves

Moltiple curves

- At least five different forwarding curve are needed:
  - EONIA
  - 1M
  - 3M
  - 6M
  - 1Y

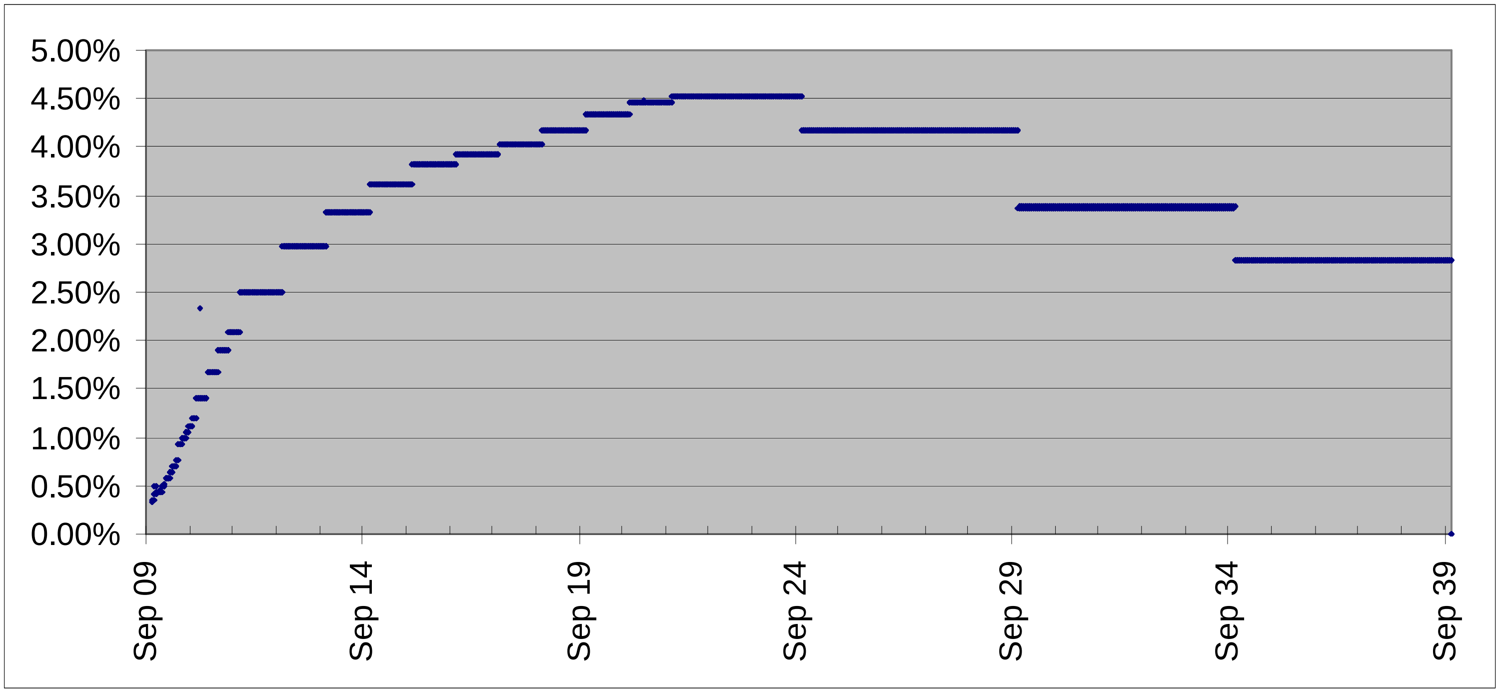
Overnight Curve

- ON, TN, SN
  - Note: EONIA is an average index, while ON, TN, SN are not average and do not have other fixings
- Spot EONIA OIS (SW, 2W, 3W, 1M, …, 12M, 15M, 18M, 21M, 2Y)
- ECB dated EONIA OIS (from spot to about 6M)
- EONIA OIS from 6M Euribor Swap minus basis (3Y-30Y)

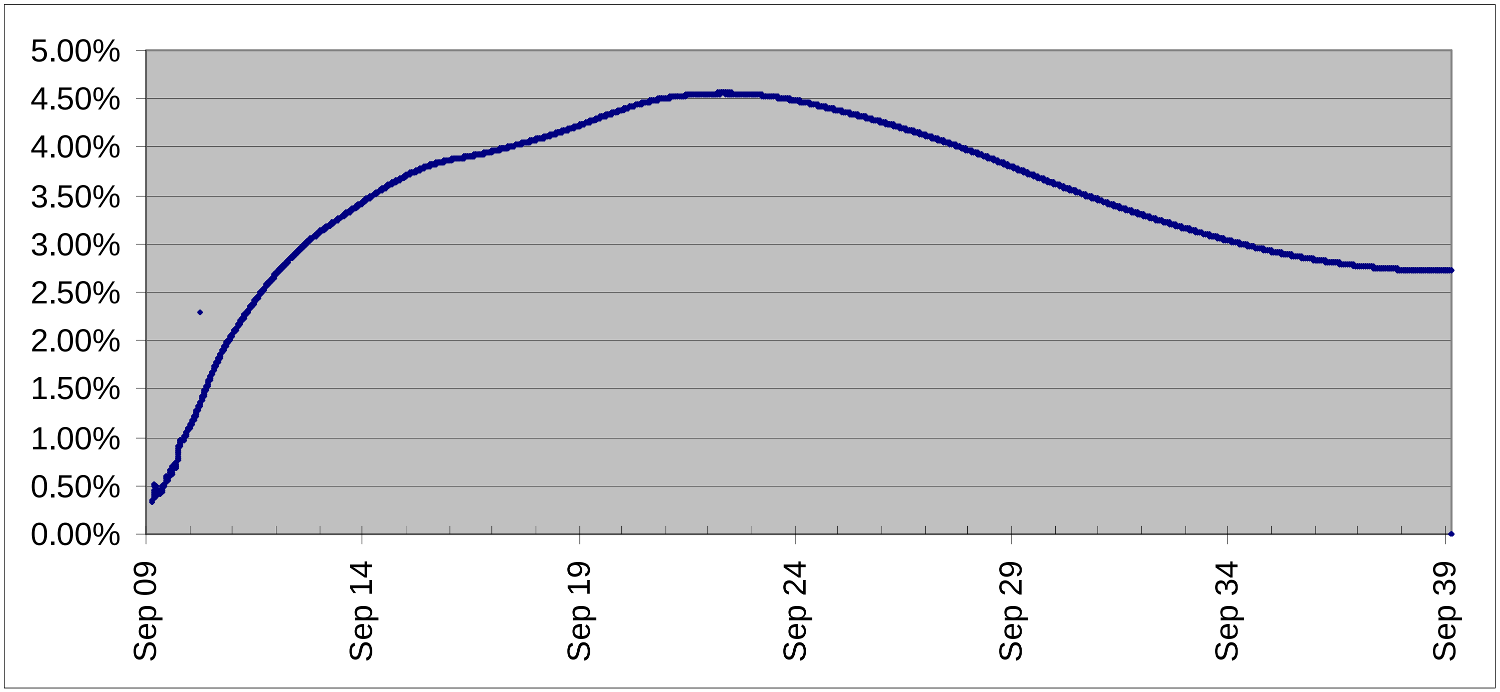
- EONIA is roughly constant between ECB dates

- It makes sense to use piecewise constant interpolation for the first 2Y, smooth interpolation later

EONIA: piecewise constant forward



EONIA: smooth forward



EONIA curve: pillars and market quotes

Rate Helpers Selected	Rate	Earliest Date	Latest Date	1.000000000
EUR_YCONRH_OND	0.3300%	Wed, 25-Nov-2009	Thu, 26-Nov-2009	0.999990833
EUR_YCONRH_TND	0.3300%	Thu, 26-Nov-2009	Fri, 27-Nov-2009	0.999981667
EUR_YCONRH_EONSW	0.3610%	Fri, 27-Nov-2009	Fri, 4-Dec-2009	0.999911479
EUR_YCONRH_EON2W	0.3830%	Fri, 27-Nov-2009	Fri, 11-Dec-2009	0.999832747
EUR_YCONRH_EON3W	0.4200%	Fri, 27-Nov-2009	Fri, 18-Dec-2009	0.999736731
EUR_YCONRH_EON1M	0.4170%	Fri, 27-Nov-2009	Mon, 28-Dec-2009	0.999622719
EUR_YCONRH_ECBOISDEC09	0.4410%	Tue, 8-Dec-2009	Wed, 20-Jan-2010	0.999342242
EUR_YCONRH_EON2M	0.4300%	Fri, 27-Nov-2009	Wed, 27-Jan-2010	0.999253600
EUR_YCONRH_ECBOISJAN10	0.4390%	Wed, 20-Jan-2010	Wed, 10-Feb-2010	0.999086392
EUR_YCONRH_EON3M	0.4430%	Fri, 27-Nov-2009	Mon, 1-Mar-2010	0.998826302
EUR_YCONRH_ECBOISFEB10	0.4960%	Wed, 10-Feb-2010	Wed, 10-Mar-2010	0.998701115
EUR_YCONRH_EON4M	0.4680%	Fri, 27-Nov-2009	Mon, 29-Mar-2010	0.998398207
EUR_YCONRH_ECBOISMAR10	0.5720%	Wed, 10-Mar-2010	Wed, 14-Apr-2010	0.998146035
EUR_YCONRH_EON5M	0.4940%	Fri, 27-Nov-2009	Tue, 27-Apr-2010	0.997913934
EUR_YCONRH_ECBOISAPR10	0.6360%	Wed, 14-Apr-2010	Wed, 12-May-2010	0.997652530
EUR_YCONRH_EON6M	0.5230%	Fri, 27-Nov-2009	Thu, 27-May-2010	0.997359084
EUR_YCONRH_ECBOISMAY10	0.7030%	Wed, 12-May-2010	Wed, 16-Jun-2010	0.996971128
EUR_YCONRH_EON7M	0.5540%	Fri, 27-Nov-2009	Mon, 28-Jun-2010	0.996714603
EUR_YCONRH_EON8M	0.6000%	Fri, 27-Nov-2009	Tue, 27-Jul-2010	0.995964610
EUR_YCONRH_EON9M	0.6440%	Fri, 27-Nov-2009	Fri, 27-Aug-2010	0.995121824
EUR_YCONRH_EON10M	0.6870%	Fri, 27-Nov-2009	Mon, 27-Sep-2010	0.994213901
EUR_YCONRH_EON11M	0.7270%	Fri, 27-Nov-2009	Wed, 27-Oct-2010	0.993282035
EUR_YCONRH_EON1Y	0.7710%	Fri, 27-Nov-2009	Mon, 29-Nov-2010	0.992183190
EUR_YCONRH_EON15M	0.9020%	Fri, 27-Nov-2009	Mon, 28-Feb-2011	0.988690125
EUR_YCONRH_EON18M	1.0260%	Fri, 27-Nov-2009	Fri, 27-May-2011	0.984671516
EUR_YCONRH_EON21M	1.1560%	Fri, 27-Nov-2009	Mon, 29-Aug-2011	0.979788993
EUR_YCONRH_EON2Y	1.2740%	Fri, 27-Nov-2009	Mon, 28-Nov-2011	0.974618037
EUR_YCONRH_EON3Y	1.6800%	Fri, 27-Nov-2009	Tue, 27-Nov-2012	0.950323041
EUR_YCONRH_EON4Y	2.0040%	Fri, 27-Nov-2009	Wed, 27-Nov-2013	0.921997491
EUR_YCONRH_EON5Y	2.2630%	Fri, 27-Nov-2009	Thu, 27-Nov-2014	0.891454784
EUR_YCONRH_EON6Y	2.4810%	Fri, 27-Nov-2009	Fri, 27-Nov-2015	0.859375379
EUR_YCONRH_EON7Y	2.6640%	Fri, 27-Nov-2009	Mon, 28-Nov-2016	0.826550941
EUR_YCONRH_EON8Y	2.8110%	Fri, 27-Nov-2009	Mon, 27-Nov-2017	0.794398043
EUR_YCONRH_EON9Y	2.9360%	Fri, 27-Nov-2009	Tue, 27-Nov-2018	0.762553129
EUR_YCONRH_EON10Y	3.0470%	Fri, 27-Nov-2009	Wed, 27-Nov-2019	0.730991169
EUR_YCONRH_EON11Y	3.1500%	Fri, 27-Nov-2009	Fri, 27-Nov-2020	0.699494335
EUR_YCONRH_EON12Y	3.2450%	Fri, 27-Nov-2009	Mon, 29-Nov-2021	0.668320955
EUR_YCONRH_EON15Y	3.4590%	Fri, 27-Nov-2009	Wed, 27-Nov-2024	0.582319099
EUR_YCONRH_EON20Y	3.6000%	Fri, 27-Nov-2009	Tue, 27-Nov-2029	0.471690237
EUR_YCONRH_EON25Y	3.5790%	Fri, 27-Nov-2009	Mon, 27-Nov-2034	0.397634242
EUR_YCONRH_EON30Y	3.5080%	Fri, 27-Nov-2009	Mon, 28-Nov-2039	0.344642390

6M Euribor curve

First key point: *select homogeneous instruments*:

- FRA 0x6 (over today and/or over tomorrow), 6x12, 12x18, (18x24)
- 6M Euribor swaps: (2Y), 3Y-10Y, 12Y, 15Y, 20Y, 25Y, 30Y, …

- Do not use deposits:
  - ON, TN, SN, SW, 1M, 2M, 3M are not homogeneous
  - 6M deposit is not in line with Euribor 6M fixing: it's not an Euribor indexed product and it is not collateralized [more on this later]

Overlapping instruments

- 1x7, 2x8, 3x9 are overlapping with 0x6 and 6x12 in the sense that do not fix a full 6M segment: their naïve introduction leads to oscillation
- Classic 1x7 FRA pricing:

$$\text{FRA1x7} = \frac{\frac{D(1M)}{D(7M)} - 1}{6M}; \qquad \frac{D(1M)}{D(7M)} = \exp\left(\int_{1M}^{7M} f(\tau) d\tau \right)$$

- The 6M Euribor market does not provide direct information about

$$D(1M) = \exp\left(- \int_0^{1M} f(\tau) d\tau \right)$$

Synthetic deposits

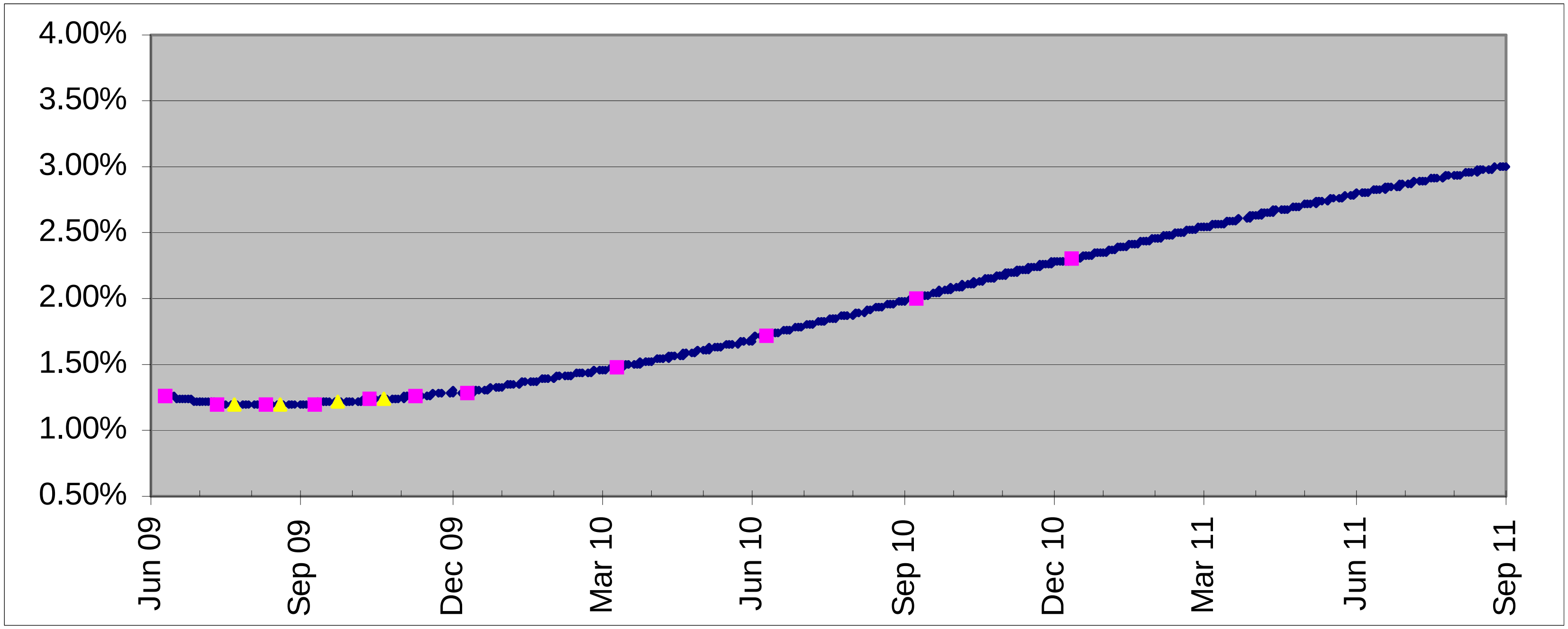
- In order to add overlapping instruments we need additional discount factors in the 0-6M region, i.e. "synthetic deposits".
- E.g. the 3M as seen on the 6M Euribor curve

- First order:
  - 6M Euribor synthetic deposits can be estimated using a parallel shift of the first 6M of the EONIA curve. The shift must match the observed basis between 0x6 and 6M EONIA OIS
- Second order:
  - Instead of a parallel shift of the first 6M of the EONIA curve allocate the overall shift in a sloped way that fits the 6M-EONIA basis term structure slope

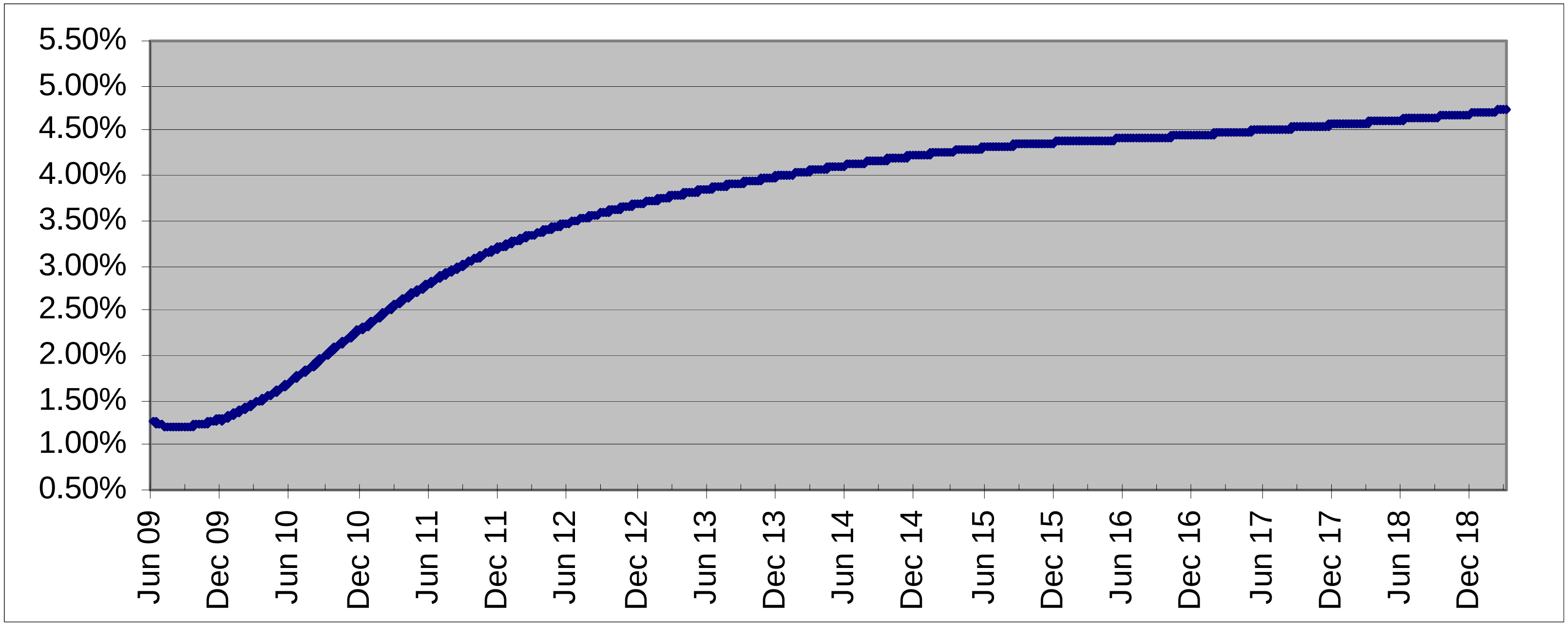
6M Euribor: pillars, market quotes, discount factors

Rate Helpers Selected	Rate		Earliest Date	Latest Date	1.000000000
EUR_YC6MRH_1MD	1.4161%	--	Wed, 8-Jul-2009	Mon, 10-Aug-2009	0.998703612
EUR_YC6MRH_2MD	1.3538%	--	Wed, 8-Jul-2009	Tue, 8-Sep-2009	0.997673949
EUR_YC6MRH_3MD	1.2983%	--	Wed, 8-Jul-2009	Thu, 8-Oct-2009	0.996693143
EUR_YC6MRH_4MD	1.2620%	--	Wed, 8-Jul-2009	Mon, 9-Nov-2009	0.995671775
EUR_YC6MRH_5MD	1.2508%	--	Wed, 8-Jul-2009	Tue, 8-Dec-2009	0.994712280
EUR_YC6MRH_TOM6F1	1.2580%	--	Thu, 9-Jul-2009	Mon, 11-Jan-2010	0.993627070
EUR_YC6MRH_1x7F	1.2050%	--	Mon, 10-Aug-2009	Wed, 10-Feb-2010	0.992714931
EUR_YC6MRH_2x8F	1.1950%	--	Tue, 8-Sep-2009	Mon, 8-Mar-2010	0.991839993
EUR_YC6MRH_3x9F	1.2060%	--	Thu, 8-Oct-2009	Thu, 8-Apr-2010	0.990777460
EUR_YC6MRH_4x10F	1.2320%	--	Mon, 9-Nov-2009	Mon, 10-May-2010	0.989632861
EUR_YC6MRH_5x11F	1.2580%	--	Tue, 8-Dec-2009	Tue, 8-Jun-2010	0.988550051
EUR_YC6MRH_6x12F	1.2850%	--	Fri, 8-Jan-2010	Thu, 8-Jul-2010	0.987339955
EUR_YC6MRH_9x15F	1.4760%	--	Thu, 8-Apr-2010	Fri, 8-Oct-2010	0.983399000
EUR_YC6MRH_12x18F	1.7260%	--	Thu, 8-Jul-2010	Mon, 10-Jan-2011	0.978688288
EUR_YC6MRH_15x21F	2.0084%	--	Fri, 8-Oct-2010	Fri, 8-Apr-2011	0.973589102
EUR_YC6MRH_18x24F	2.2970%	--	Mon, 10-Jan-2011	Mon, 11-Jul-2011	0.967453625
EUR_YC6MRH_AB6E3Y	2.1260%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2012	0.938659728
EUR_YC6MRH_AB6E4Y	2.4920%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2013	0.905555359
EUR_YC6MRH_AB6E5Y	2.7760%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jul-2014	0.870597330
EUR_YC6MRH_AB6E6Y	3.0030%	0.0000%	Wed, 8-Jul-2009	Wed, 8-Jul-2015	0.834923717
EUR_YC6MRH_AB6E7Y	3.1880%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2016	0.799259260
EUR_YC6MRH_AB6E8Y	3.3350%	0.0000%	Wed, 8-Jul-2009	Mon, 10-Jul-2017	0.764359144
EUR_YC6MRH_AB6E9Y	3.4580%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2018	0.730470147
EUR_YC6MRH_AB6E10Y	3.5660%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2019	0.697249973
EUR_YC6MRH_AB6E12Y	3.7550%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2021	0.632493436
EUR_YC6MRH_AB6E15Y	3.9570%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2024	0.544688772
EUR_YC6MRH_AB6E20Y	4.1010%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2029	0.431063815
EUR_YC6MRH_AB6E25Y	4.0760%	0.0000%	Wed, 8-Jul-2009	Mon, 10-Jul-2034	0.356242899
EUR_YC6MRH_AB6E30Y	4.0190%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2039	0.300678089
EUR_YC6MRH_AB6E35Y	3.9450%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2044	0.259339397
EUR_YC6MRH_AB6E40Y	3.8710%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2049	0.226876136
EUR_YC6MRH_AB6E50Y	3.7940%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jul-2059	0.168728506
EUR_YC6MRH_AB6E60Y	3.7350%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2069	0.127557323

6M Euribor curve



6M Euribor curve (2)



3M Euribor curve

Homogeneous instrument selection:

- 0x3 FRA (other FRAs are less liquid than futures)
- Futures strip (usually 8 contracts + optional first serial)
- 3M Euribor swaps (from basis) 3Y-10Y, 12Y, 15Y, 20Y, 25Y, 30Y, …

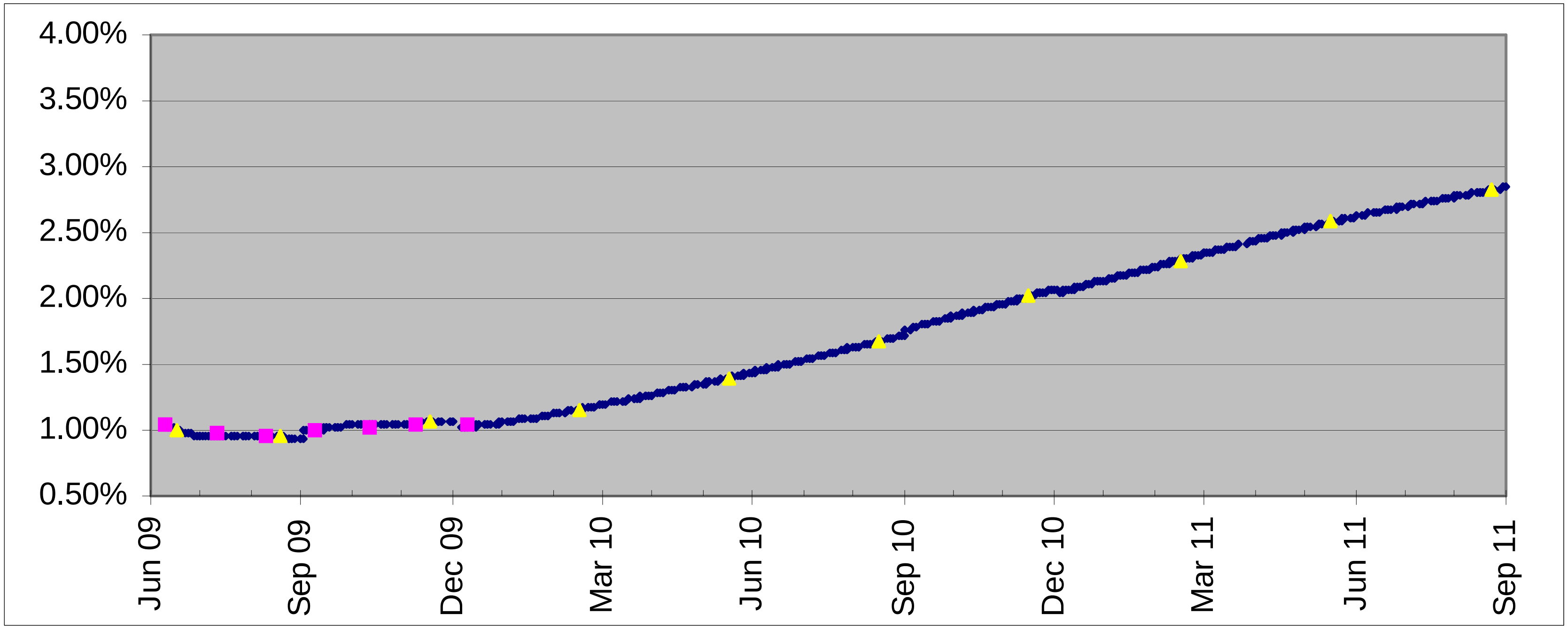
- Second key point: 3M Euribor swap rates are obtained from the same maturity 6M Euribor swap rates minus same maturity 3M/6M basis swaps

- Again
  - Do no use deposits
  - Use synthetic deposits (0x3 is always overlapping with futures)

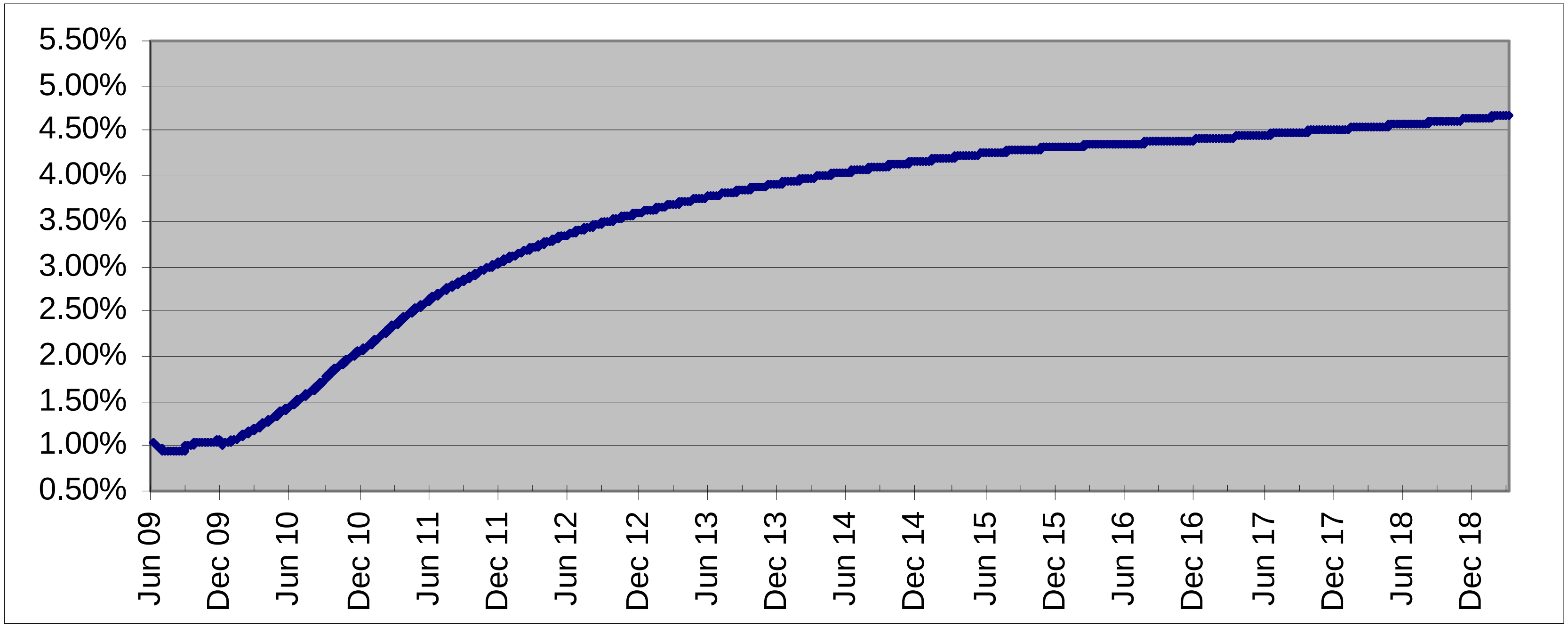
3M Euribor: pillars, market quotes, discount factors

Rate Helpers Selected	Rate		Earliest Date	Latest Date	1.000000000
EUR_YC3MRH_2WD	1.2183%	--	Wed, 8-Jul-2009	Wed, 22-Jul-2009	0.999526446
EUR_YC3MRH_1MD	1.1196%	--	Wed, 8-Jul-2009	Mon, 10-Aug-2009	0.998974782
EUR_YC3MRH_2MD	1.0607%	--	Wed, 8-Jul-2009	Tue, 8-Sep-2009	0.998176561
EUR_YC3MRH_TOM3F1	1.0370%	--	Thu, 9-Jul-2009	Fri, 9-Oct-2009	0.997322184
EUR_YC3MRH_FUT3MN9	1.0075%	0.0000%	Wed, 15-Jul-2009	Thu, 15-Oct-2009	0.997190693
EUR_YC3MRH_FUT3MU9	0.9471%	0.0004%	Wed, 16-Sep-2009	Wed, 16-Dec-2009	0.995560088
EUR_YC3MRH_FUT3MZ9	1.0562%	0.0013%	Wed, 16-Dec-2009	Tue, 16-Mar-2010	0.993062730
EUR_YC3MRH_FUT3MH0	1.1600%	0.0025%	Wed, 17-Mar-2010	Thu, 17-Jun-2010	0.990098576
EUR_YC3MRH_FUT3MM0	1.3985%	0.0040%	Wed, 16-Jun-2010	Thu, 16-Sep-2010	0.986607501
EUR_YC3MRH_FUT3MU0	1.6766%	0.0059%	Wed, 15-Sep-2010	Wed, 15-Dec-2010	0.982485576
EUR_YC3MRH_FUT3MZ0	2.0144%	0.0081%	Wed, 15-Dec-2010	Tue, 15-Mar-2011	0.977637808
EUR_YC3MRH_FUT3MH1	2.2918%	0.0107%	Wed, 16-Mar-2011	Thu, 16-Jun-2011	0.971887801
EUR_YC3MRH_FUT3MM1	2.5764%	0.0136%	Wed, 15-Jun-2011	Thu, 15-Sep-2011	0.965595905
EUR_YC3MRH_AB3E3Y	2.0100%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2012	0.941842233
EUR_YC3MRH_AB3E4Y	2.3960%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2013	0.908914516
EUR_YC3MRH_AB3E5Y	2.6940%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jul-2014	0.874012742
EUR_YC3MRH_AB3E6Y	2.9310%	0.0000%	Wed, 8-Jul-2009	Wed, 8-Jul-2015	0.838338290
EUR_YC3MRH_AB3E7Y	3.1230%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2016	0.802666759
EUR_YC3MRH_AB3E8Y	3.2760%	0.0000%	Wed, 8-Jul-2009	Mon, 10-Jul-2017	0.767698882
EUR_YC3MRH_AB3E9Y	3.4040%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2018	0.733707818
EUR_YC3MRH_AB3E10Y	3.5160%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2019	0.700381926
EUR_YC3MRH_AB3E12Y	3.7120%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2021	0.635311790
EUR_YC3MRH_AB3E15Y	3.9200%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2024	0.547174480
EUR_YC3MRH_AB3E20Y	4.0700%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2029	0.433077252
EUR_YC3MRH_AB3E25Y	4.0490%	0.0000%	Wed, 8-Jul-2009	Mon, 10-Jul-2034	0.357872488
EUR_YC3MRH_AB3E30Y	3.9940%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2039	0.302130119
EUR_YC3MRH_AB3EBASIS35Y	3.9200%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2044	0.260893433
EUR_YC3MRH_AB3EBASIS40Y	3.8460%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2049	0.228482302
EUR_YC3MRH_AB3EBASIS50Y	3.7690%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jul-2059	0.170286910
EUR_YC3MRH_AB3EBASIS60Y	3.7100%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2069	0.128986064

3M Euribor curve



3M Euribor curve (2)



1M Euribor curve

Homogeneous instrument selection:

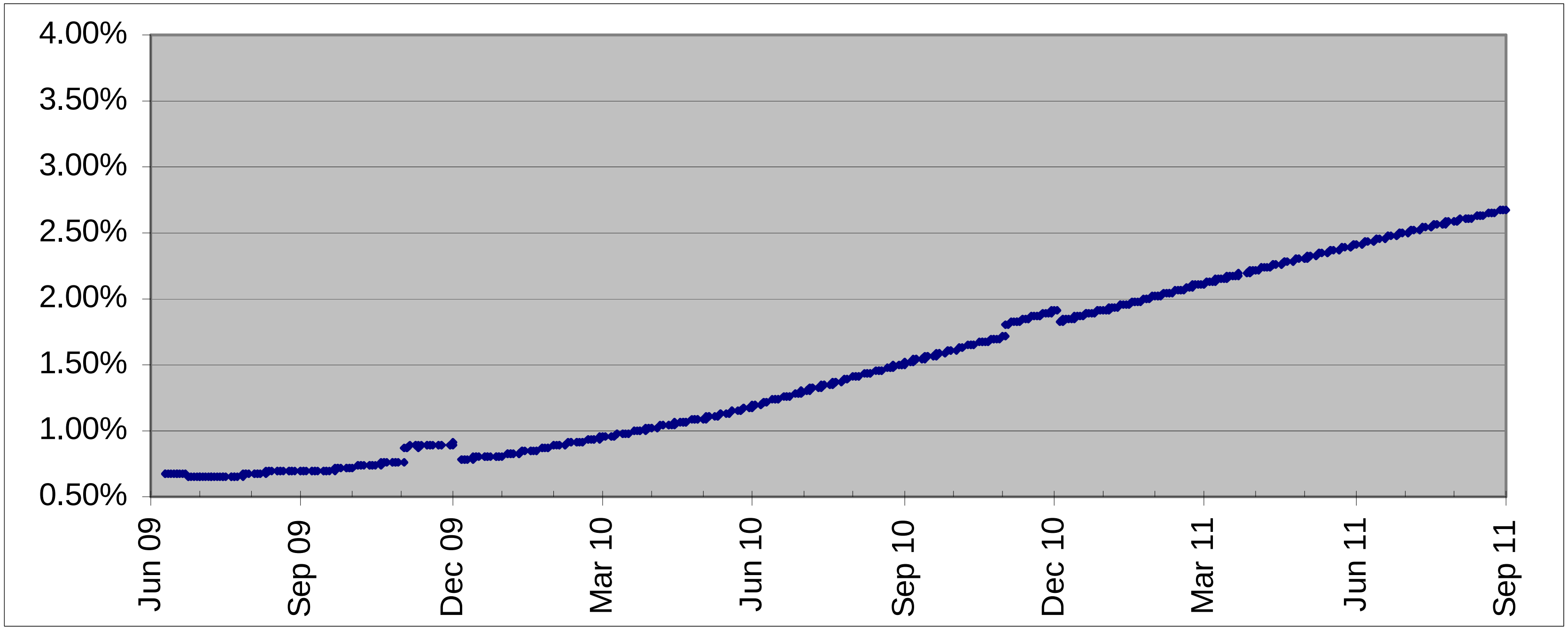
- Money market monthly swaps (maturities ranging in 2M-12M, fixed rate vs 1M Euribor)
- 1M Euribor Swap (from basis) 2Y-10Y, 12Y, 15Y, 20Y, 25Y, 30Y, …

- There is not the 0x1 FRA on the market... use the fixing and/or play with the basis term structure
- No overlapping instruments -> no need for synthetic deposits, but it's possible to use them for greater curve granularity

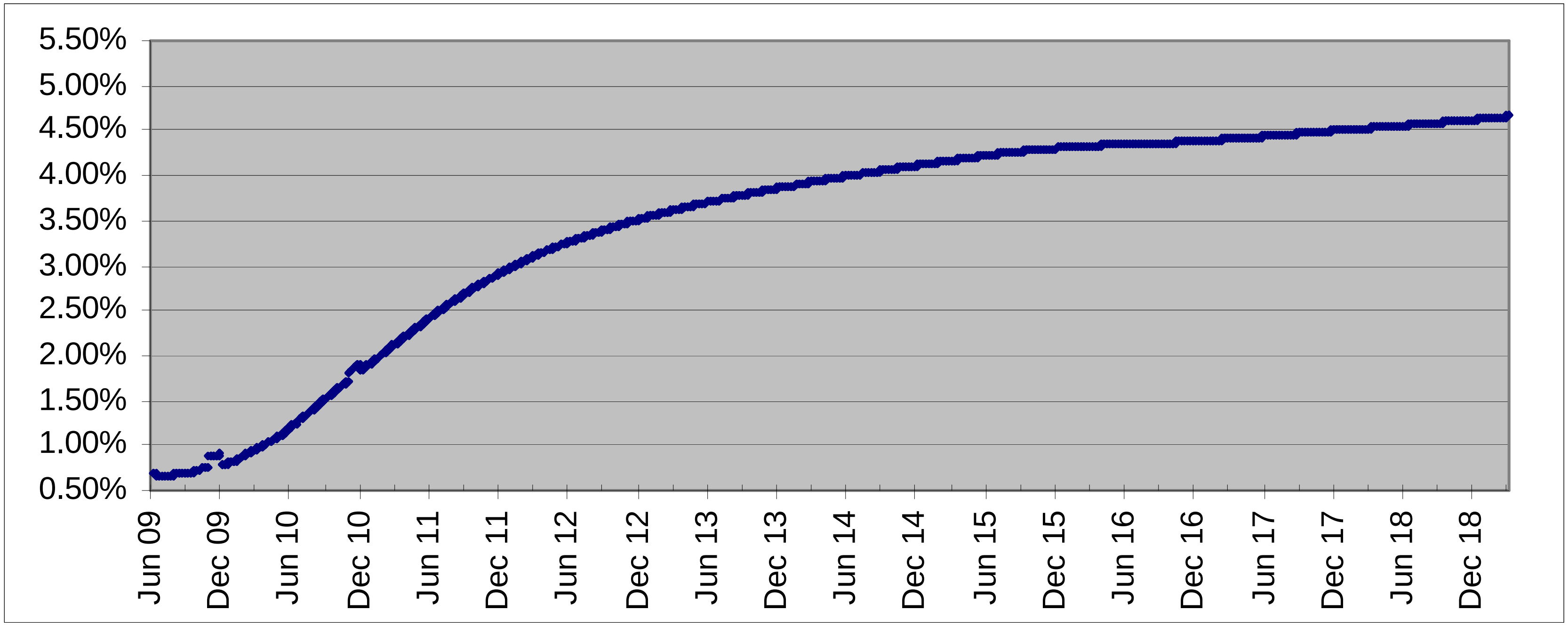
1M Euribor: pillars, market quotes, discount factors

Rate Helpers Selected	Rate		Earliest Date	Latest Date	1.000000000
EUR_YC1MRH_1MD	0.6840%	--	Wed, 8-Jul-2009	Mon, 10-Aug-2009	0.999373393
EUR_YC1MRH_2X1S	0.6650%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Sep-2009	0.998856032
EUR_YC1MRH_3X1S	0.6720%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Oct-2009	0.998285611
EUR_YC1MRH_4X1S	0.6780%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Nov-2009	0.997670108
EUR_YC1MRH_5X1S	0.6900%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Dec-2009	0.997076074
EUR_YC1MRH_6X1S	0.7230%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jan-2010	0.996417113
EUR_YC1MRH_7X1S	0.7330%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Feb-2010	0.995740215
EUR_YC1MRH_8X1S	0.7450%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Mar-2010	0.995095122
EUR_YC1MRH_9X1S	0.7630%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Apr-2010	0.994324886
EUR_YC1MRH_10X1S	0.7850%	0.0000%	Wed, 8-Jul-2009	Mon, 10-May-2010	0.993470276
EUR_YC1MRH_11X1S	0.8080%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jun-2010	0.992635689
EUR_YC1MRH_12X1S	0.8340%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2010	0.991713443
EUR_YC1MRH_AB1EBASIS2Y	1.3350%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2011	0.973933729
EUR_YC1MRH_AB1EBASIS3Y	1.8610%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2012	0.945941159
EUR_YC1MRH_AB1EBASIS4Y	2.2740%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2013	0.913196722
EUR_YC1MRH_AB1EBASIS5Y	2.5910%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jul-2014	0.878314495
EUR_YC1MRH_AB1EBASIS6Y	2.8420%	0.0000%	Wed, 8-Jul-2009	Wed, 8-Jul-2015	0.842563637
EUR_YC1MRH_AB1EBASIS7Y	3.0470%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2016	0.806617062
EUR_YC1MRH_AB1EBASIS8Y	3.2090%	0.0000%	Wed, 8-Jul-2009	Mon, 10-Jul-2017	0.771433274
EUR_YC1MRH_AB1EBASIS9Y	3.3440%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2018	0.737225896
EUR_YC1MRH_AB1EBASIS10Y	3.4620%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2019	0.703654305
EUR_YC1MRH_AB1EBASIS12Y	3.6660%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2021	0.638206538
EUR_YC1MRH_AB1EBASIS15Y	3.8820%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2024	0.549549084
EUR_YC1MRH_AB1EBASIS20Y	4.0400%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2029	0.434767426
EUR_YC1MRH_AB1EBASIS25Y	4.0240%	0.0000%	Wed, 8-Jul-2009	Mon, 10-Jul-2034	0.359078175
EUR_YC1MRH_AB1EBASIS30Y	3.9720%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2039	0.303051598
EUR_YC1MRH_AB1EBASIS35Y	3.8980%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2044	0.261976994
EUR_YC1MRH_AB1EBASIS40Y	3.8240%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2049	0.229667741
EUR_YC1MRH_AB1EBASIS50Y	3.7470%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jul-2059	0.171510047
EUR_YC1MRH_AB1EBASIS60Y	3.6880%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2069	0.130147207

1M Euribor curve



1M Euribor curve (2)



1Y Euribor curve

Homogeneous instrument selection:

- 12x24 FRA
- 1Y Euribor swap (from basis) 3Y-10Y, 12Y, 15Y, 20Y, 25Y, 30Y, …

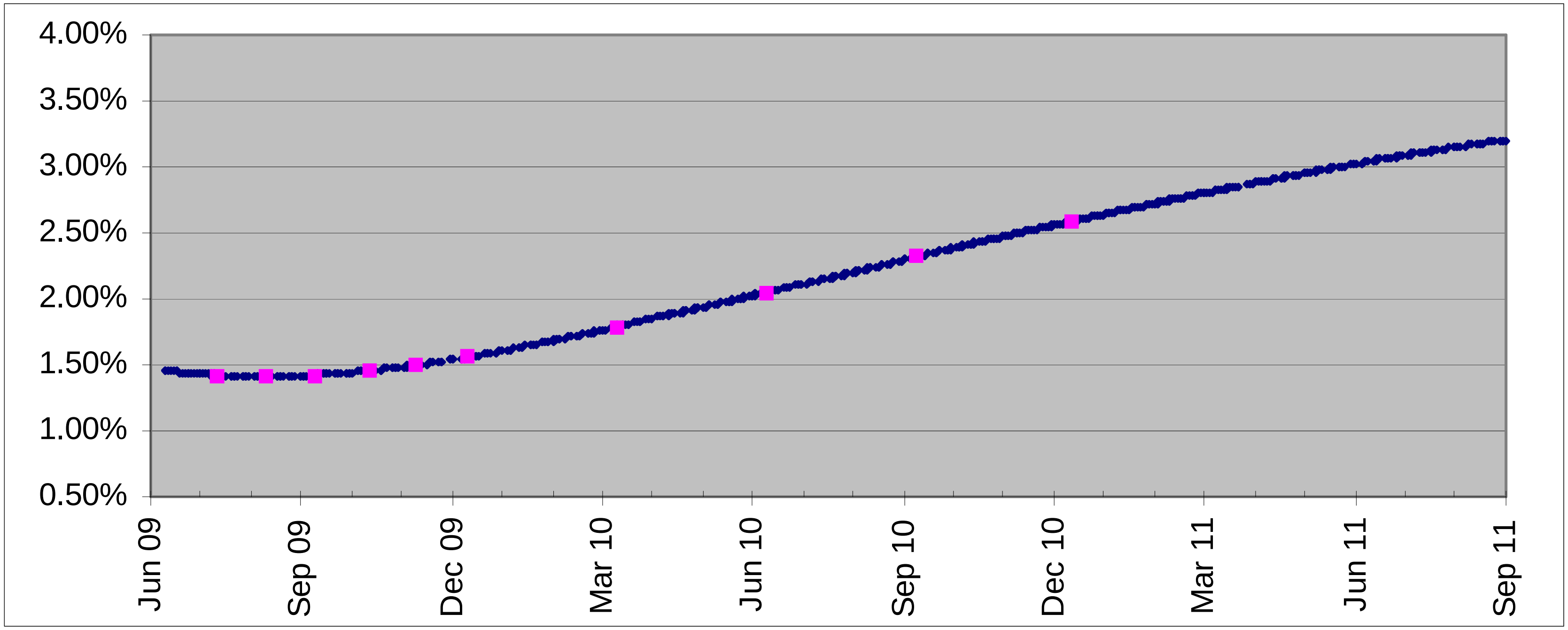
- There is not the 0x12 FRA on the market... use the fixing and/or play with the basis term structure

- Using only 0x12, 12x24 is too loose for market makers and results in unreliable intermediate 6x18
- Use 1Y/6M basis term structure to interpolate 3x15, 6x18, 9x21 (and 1x13, 2x14, etc)

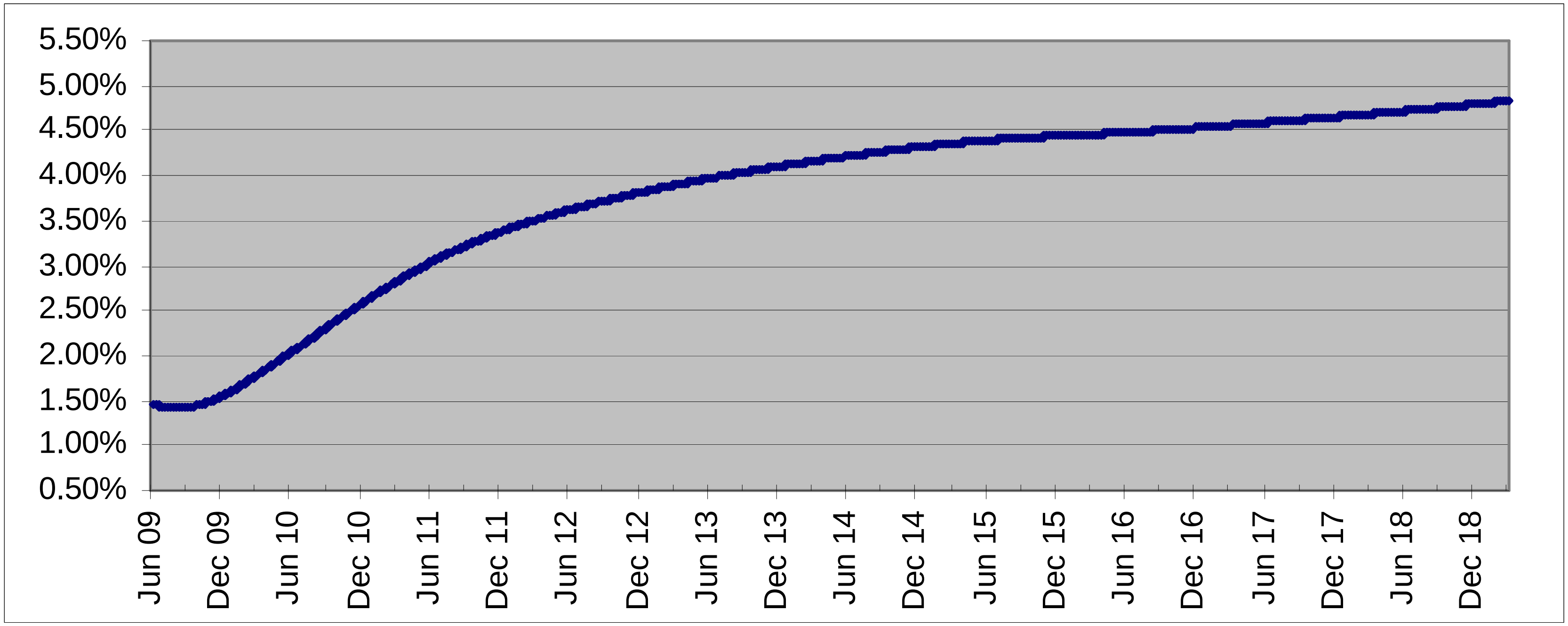
1Y Euribor: pillars, market quotes, discount factors

Rate Helpers Selected	Rate		Earliest Date	Latest Date	1.000000000
EUR_YC1YRH_1MD	1.9789%	--	Wed, 8-Jul-2009	Mon, 10-Aug-2009	0.998189307
EUR_YC1YRH_2MD	1.8867%	--	Wed, 8-Jul-2009	Tue, 8-Sep-2009	0.996761196
EUR_YC1YRH_3MD	1.7666%	--	Wed, 8-Jul-2009	Thu, 8-Oct-2009	0.995505563
EUR_YC1YRH_6MD	1.5686%	--	Wed, 8-Jul-2009	Fri, 8-Jan-2010	0.992046407
EUR_YC1YRH_9MD	1.4660%	--	Wed, 8-Jul-2009	Thu, 8-Apr-2010	0.988965224
EUR_YC1YRH_1YD	1.4560%	--	Wed, 8-Jul-2009	Thu, 8-Jul-2010	0.985452531
EUR_YC1YRH_1x13F	1.4204%	--	Mon, 10-Aug-2009	Tue, 10-Aug-2010	0.984018529
EUR_YC1YRH_2x14F	1.4084%	--	Tue, 8-Sep-2009	Wed, 8-Sep-2010	0.982728122
EUR_YC1YRH_3x15F	1.4228%	--	Thu, 8-Oct-2009	Fri, 8-Oct-2010	0.981348597
EUR_YC1YRH_6x18F	1.5599%	--	Fri, 8-Jan-2010	Mon, 10-Jan-2011	0.976517569
EUR_YC1YRH_9x21F	1.7888%	--	Thu, 8-Apr-2010	Fri, 8-Apr-2011	0.971348075
EUR_YC1YRH_12x24F	2.0470%	--	Thu, 8-Jul-2010	Fri, 8-Jul-2011	0.965415992
EUR_YC1YRH_15x27F	2.3231%	--	Fri, 8-Oct-2010	Mon, 10-Oct-2011	0.958645347
EUR_YC1YRH_18x30F	2.5891%	--	Mon, 10-Jan-2011	Tue, 10-Jan-2012	0.951538706
EUR_YC1YRH_AB12EBASIS3Y	2.2010%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2012	0.936394100
EUR_YC1YRH_AB12EBASIS4Y	2.5510%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2013	0.903300392
EUR_YC1YRH_AB12EBASIS5Y	2.8260%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jul-2014	0.868336891
EUR_YC1YRH_AB12EBASIS6Y	3.0460%	0.0000%	Wed, 8-Jul-2009	Wed, 8-Jul-2015	0.832722385
EUR_YC1YRH_AB12EBASIS7Y	3.2260%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2016	0.797122139
EUR_YC1YRH_AB12EBASIS8Y	3.3690%	0.0000%	Wed, 8-Jul-2009	Mon, 10-Jul-2017	0.762304390
EUR_YC1YRH_AB12EBASIS9Y	3.4890%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2018	0.728491475
EUR_YC1YRH_AB12EBASIS10Y	3.5950%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2019	0.695315561
EUR_YC1YRH_AB12EBASIS12Y	3.7800%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2021	0.630749929
EUR_YC1YRH_AB12EBASIS15Y	3.9780%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2024	0.543211294
EUR_YC1YRH_AB12EBASIS20Y	4.1190%	0.0000%	Wed, 8-Jul-2009	Mon, 9-Jul-2029	0.429818737
EUR_YC1YRH_AB12EBASIS25Y	4.0930%	0.0000%	Wed, 8-Jul-2009	Mon, 10-Jul-2034	0.355049339
EUR_YC1YRH_AB12EBASIS30Y	4.0350%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2039	0.299587957
EUR_YC1YRH_AB12EBASIS35Y	3.9610%	0.0000%	Wed, 8-Jul-2009	Fri, 8-Jul-2044	0.258213019
EUR_YC1YRH_AB12EBASIS40Y	3.8870%	0.0000%	Wed, 8-Jul-2009	Thu, 8-Jul-2049	0.225739564
EUR_YC1YRH_AB12EBASIS50Y	3.8100%	0.0000%	Wed, 8-Jul-2009	Tue, 8-Jul-2059	0.167657933
EUR_YC1YRH_AB12EBASIS60Y	3.7510%	0.0000%	Wed, 8-Jul-2009	Mon, 8-Jul-2069	0.126593622

1Y Euribor curve



1Y Euribor curve (2)



Curve comparison: FRA and futures

	Euribor 1M	Euribor 3M	Euribor 6M	Euribor 1Y
U9	99.2970	99.0429	98.7930	
Z9	99.1277	98.9237		
H0	99.0490	98.8150		
M0	98.8264	98.5765		
U0		98.3034		
Z0		97.9656		
H1		97.6932		
M1		97.4111		
FRA TODAY		1.0366%	1.2512%	
FRA TOM		1.0300%	1.2500%	
FRA1x		0.9619%	1.2140%	1.4198%
FRA3x		1.0229%	1.2220%	1.4359%
FRA6x		1.0540%	1.3080%	1.5834%
FRA9x			1.4963%	1.8085%
FRA12x			1.7510%	2.0730%
FRA15x			2.0434%	2.3503%
FRA18x			2.3260%	2.6223%

Curve comparison: swaps

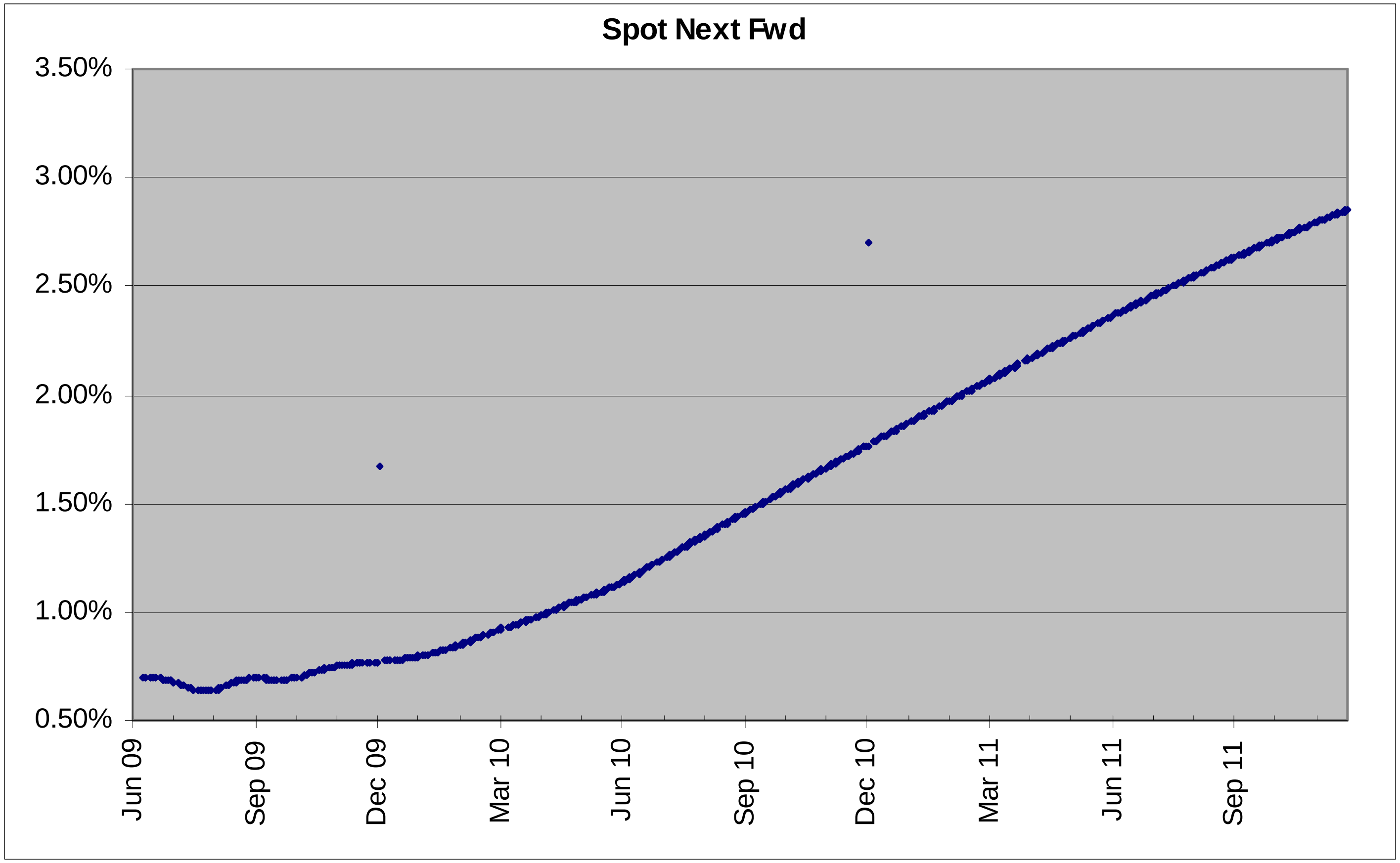
	Euribor 1M	Euribor 3M	Euribor 6M	Euribor 1Y
3Y	1.8870%	2.0360%	2.1510%	2.2200%
4Y	2.2930%	2.4150%	2.5100%	2.5650%
5Y	2.6030%	2.7070%	2.7880%	2.8340%
6Y	2.8540%	2.9420%	3.0140%	3.0540%
7Y	3.0580%	3.1350%	3.1990%	3.2340%
8Y	3.2210%	3.2890%	3.3470%	3.3790%
9Y	3.3560%	3.4160%	3.4700%	3.4990%
10Y	3.4730%	3.5280%	3.5770%	3.6040%
12Y	3.6770%	3.7230%	3.7660%	3.7890%
15Y	3.8930%	3.9310%	3.9680%	3.9880%
20Y	4.0500%	4.0800%	4.1110%	4.1280%
25Y	4.0340%	4.0590%	4.0860%	4.1020%
30Y	3.9820%	4.0040%	4.0290%	4.0440%
40Y	3.8340%	3.8560%	3.8810%	3.8960%
50Y	3.7570%	3.7790%	3.8040%	3.8190%
60Y	3.6980%	3.7200%	3.7450%	3.7600%

Focus lens

- We have plotted (simple compounding) FRA rates since this is what traders are interested in
- What about instantaneous (continuous compounding) forward rates?
- On the one day scale continuous compounding forward rates and simple compounding (i.e. ON) rates are equivalent

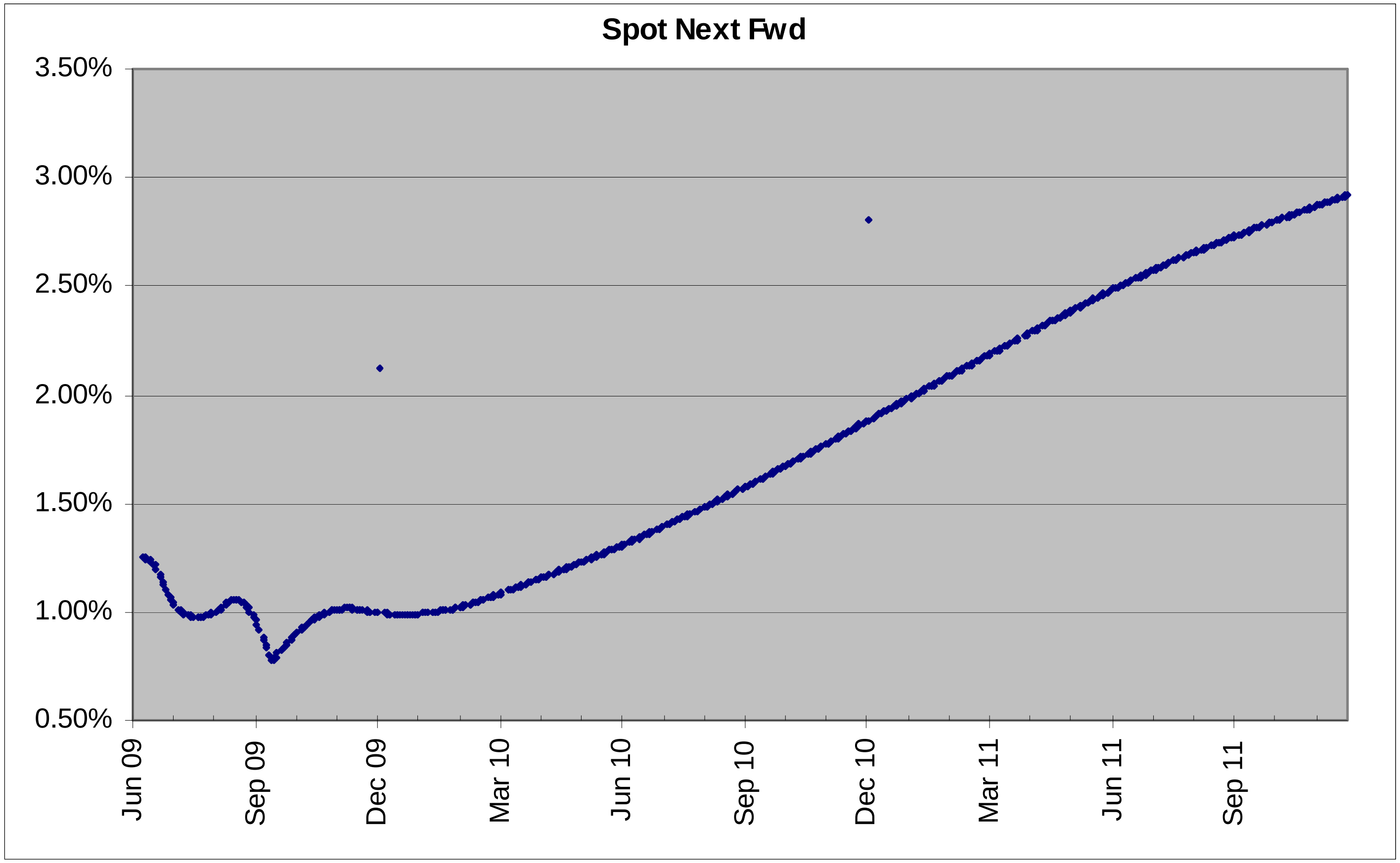
ON rates as seen on the 1M Euribor curve

- Note the TOYs



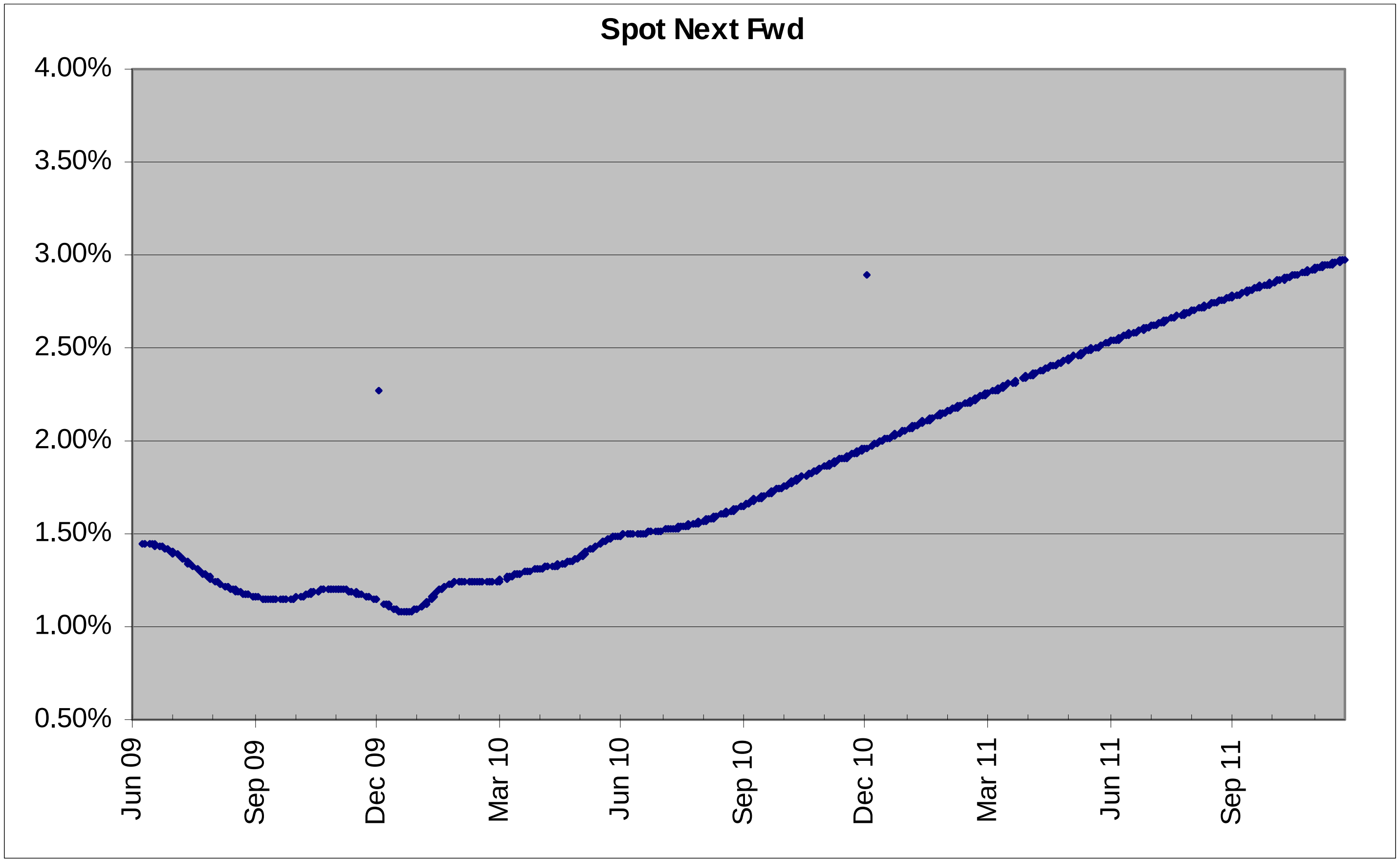
ON rates as seen on the 3M Euribor curve

- Note the TOYs



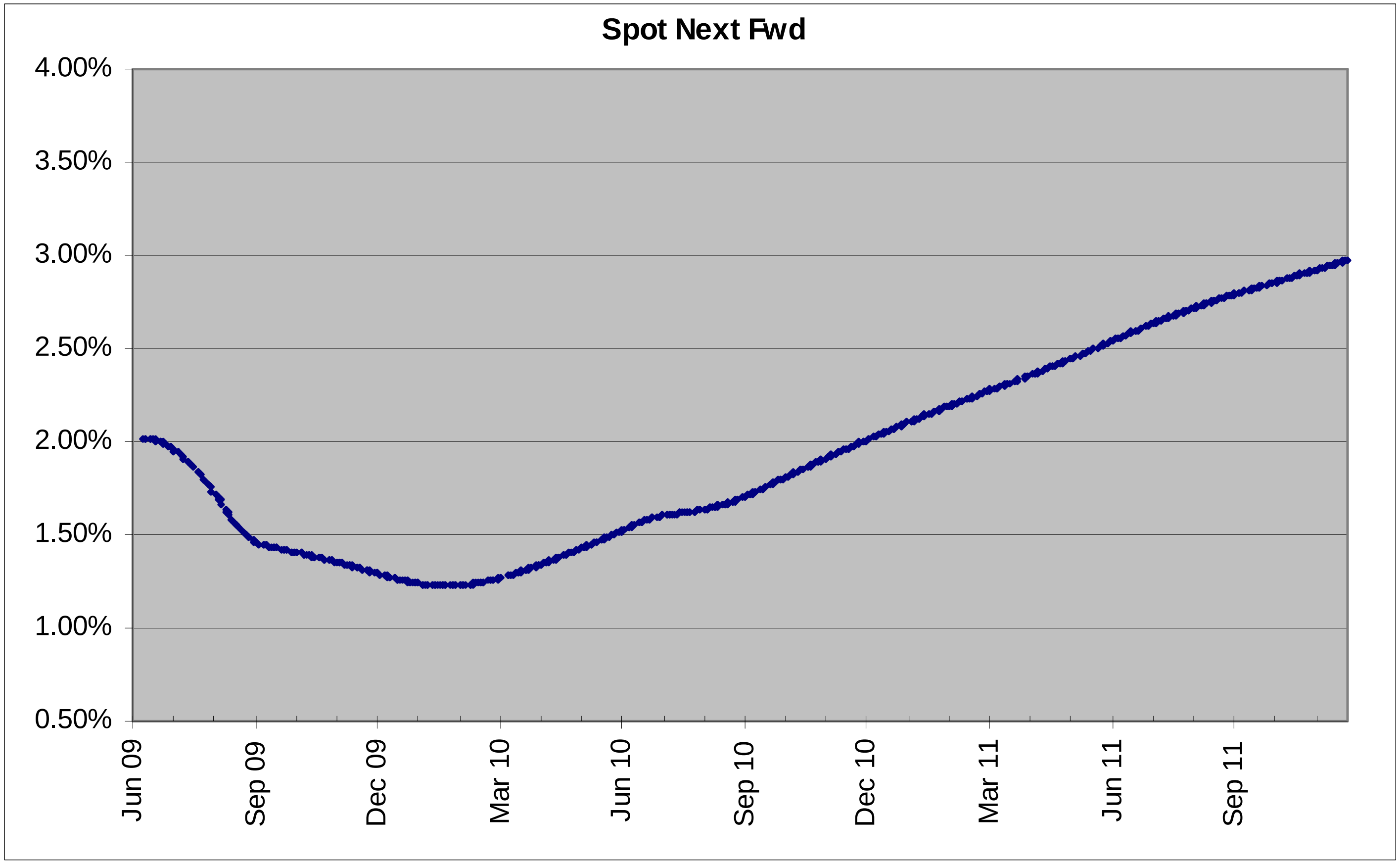
ON rates as seen on the 6M Euribor curve

- Note the TOYs



ON rates as seen on the 1Y Euribor curve

- No TOYs here



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Rate curves for forward Euribor estimation and CSA-discounting

7. Discounting Rate Curve

Discounting curve? What do you mean/want?

- Two identical future cashflows must have the same present value: we need an unique discounting curve.
- We have bootstrapped each forwarding curve using the forwarding curve itself also for discounting swap cashflows. Something is flawed here, at least when swaps are bootstrapped
- The discounting curve should represent the funding level implicit in whatever hedging strategy. What is the funding level?

Discounting and collateralization

- What are swap rates? They are rates tradable between collateralized counterparties.

- Capital market collateralization between two counterparties is the bilateral obligation to secure by liquid assets (such as cash or securities) the outstanding NPV of the overall trading book. This assets are called margin. The margin pledged by the borrower are legally in the lender possession or subject to seizure in the event of default

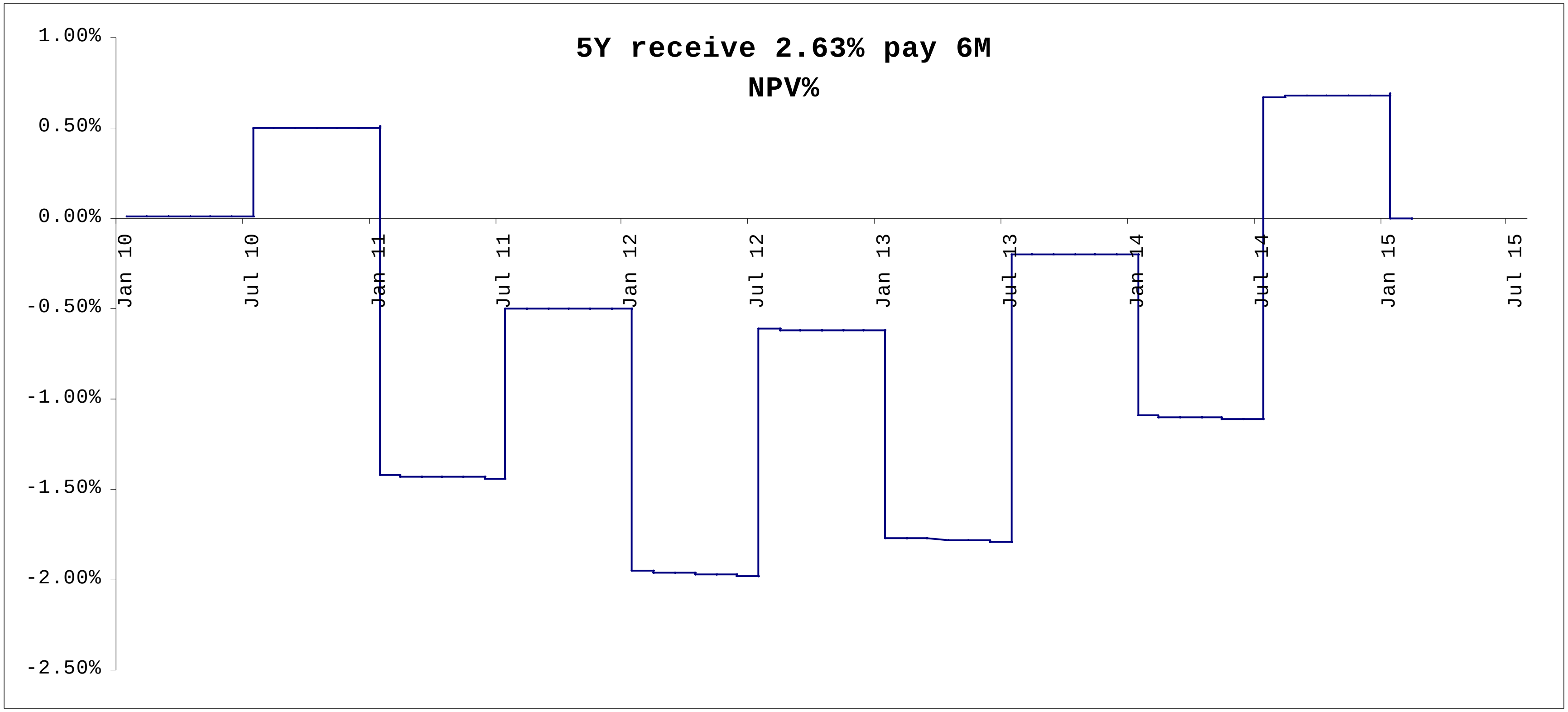
- The collateral margin earns the overnight rate: ***the overnight curve is the discounting curve for collateralized transactions***

- Using the the same rationale: uncollateralized transactions should be discounted by each financial institution using its own capital market funding rates

What about counterparty credit risk?

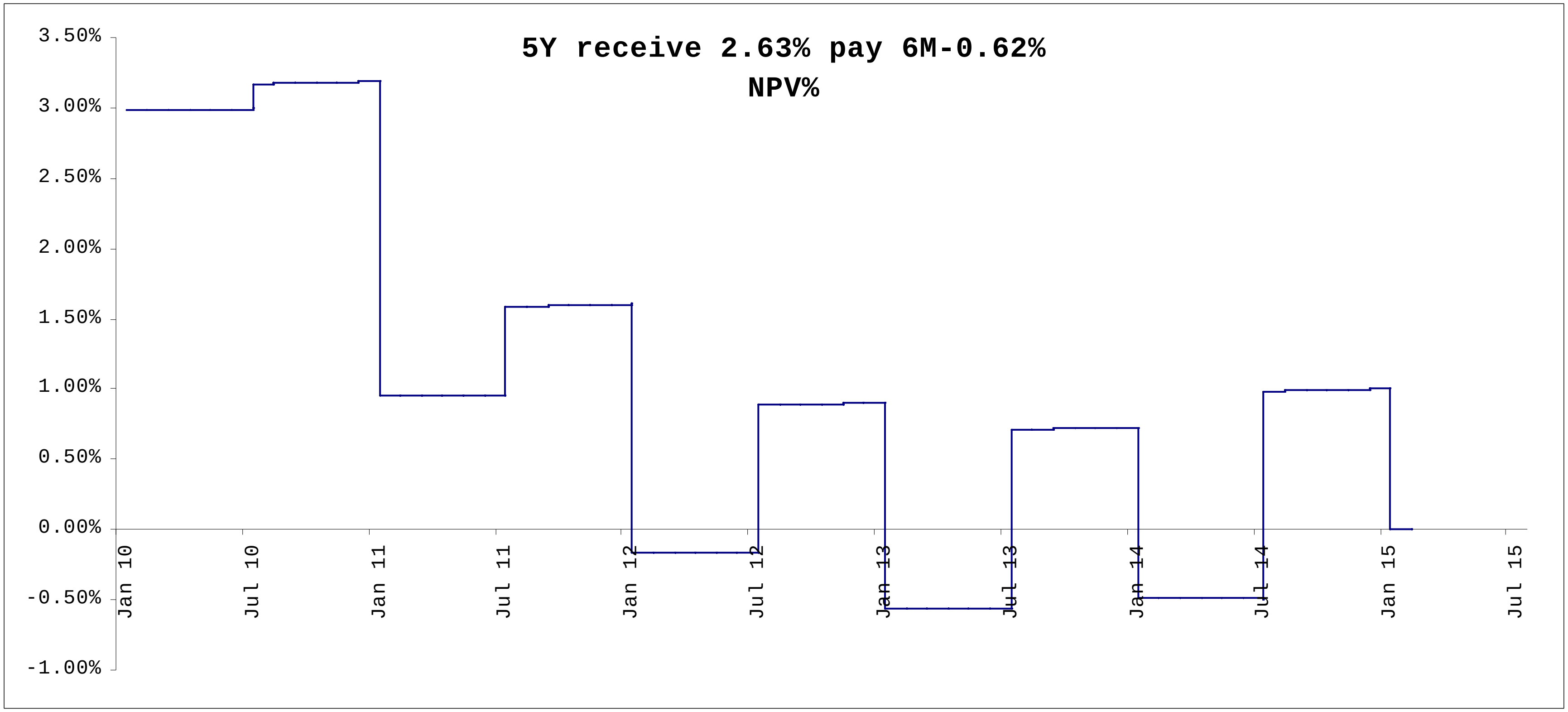
- Collateralized transactions have negligible residual credit risk. After all that's what collateralization was created for!
- Uncollateralized transactions have credit risk which <u>must</u> be accounted for, but this has little to do with the liquidity/funding issue.

5Y Receiver Swap 2.63% 6M flat
NPV evolution (Deterministic Curve)



- Average NPV -0.64%, positive cash balance: borrowing

Asset Swap 5Y bond 2.63% 103.00
NPV evolution (Deterministic Curve)



- Average NPV 1.02%, negative cash balance: lending

Forwarding and discounting rate curves: a recipe

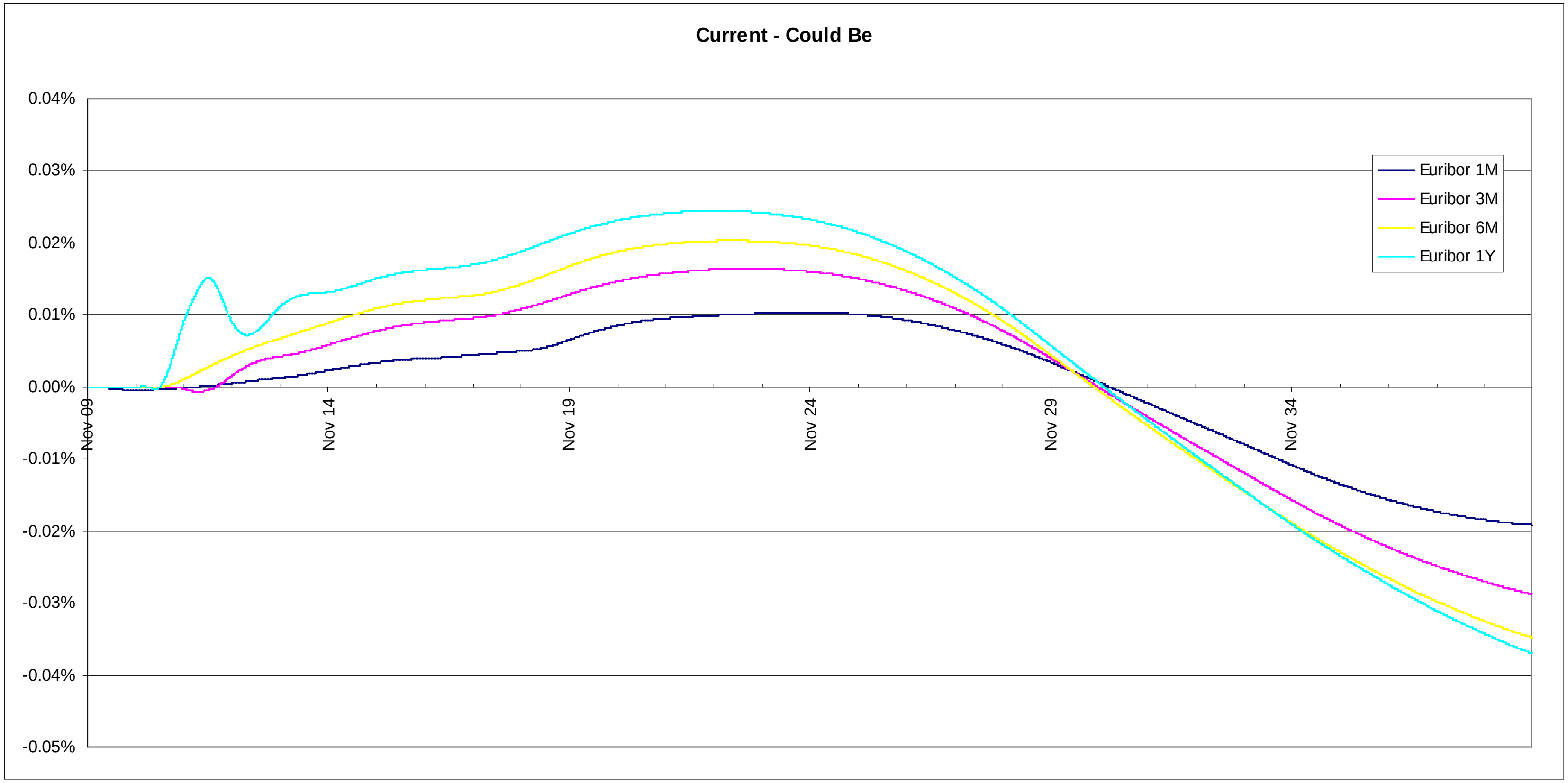
1. build the EONIA curve using your preferred procedure; this is the EONIA forwarding curve and the discount curve for collateralized transactions

2. select different sets of collateralized vanilla interest rate instruments traded on the market, each set homogeneous in the underlying Euribor rate

3. build separated forwarding curves using the selected instruments in the bootstrapping algorithm; use the EONIA curve to exogenously discount any cashflow

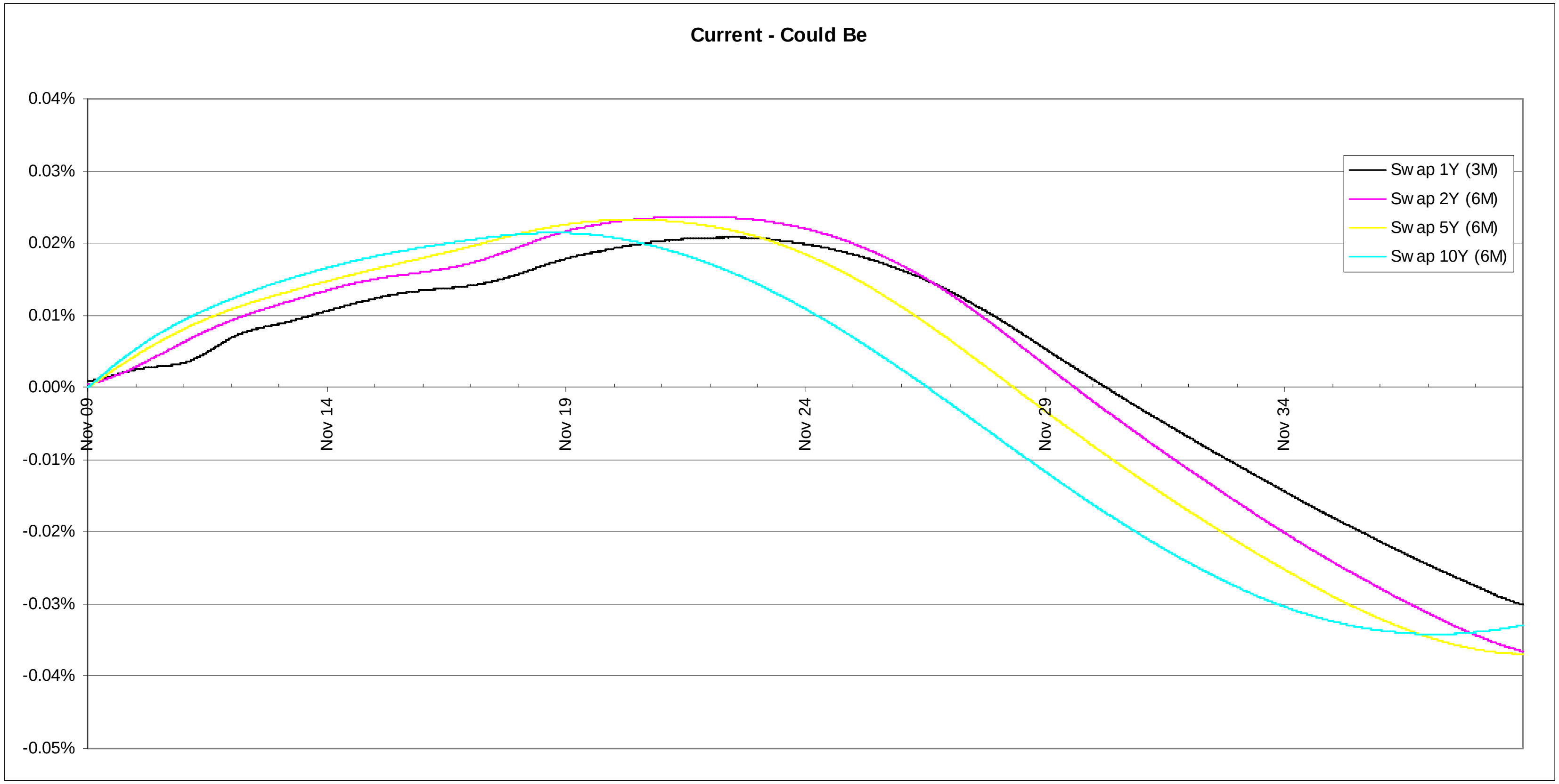
The impact of exogenous EONIA discounting

forward Euribor



The impact of exogenous EONIA discounting (2)

forward Swap



Rate curves for forward Euribor estimation and CSA-discounting

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