Risk simulations for a bond in QuantLibXL

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StatPro asset coverage

- Equities, equity indexes, and funds
- Equity options: vanilla options, digital and barrier options, ...
- Interest-rate products: IR swaps, IR caps and floors, swaptions, constant-maturity swaps, overnight swaps, ...
- Bonds: fixed-rate, floating-rate, reverse floaters, CMS bonds, ...
- Equity-linked bonds: bonds with embedded options, Himalaya, ...
- Inflation-linked bonds, inflation derivatives
- Foreign-exchange instruments: cross-currency swaps, options, ...
- Credit derivatives: credit-default swaps, ...
- Mortgage-backed securities, asset-backed securities, ...
- ...

Executive summary

- Consider a simple non-trivial instrument: the fixed-rate coupon bond
- Consider in details the yield-curve and credit-spread risk factors
- Show from the beginning to the end the computation of risk figures using tools available in QuantLib

Talk outline

- Part I: QuantLib tools for pricing (buy side)

- Part II: QuantLib tools for risk management

Examples using the QuantLib Addin and QuantLibXL (ver. 1.0.1)

Part I: Pricing tools

1. Bootstrap of a yield curve from quoted swap rates

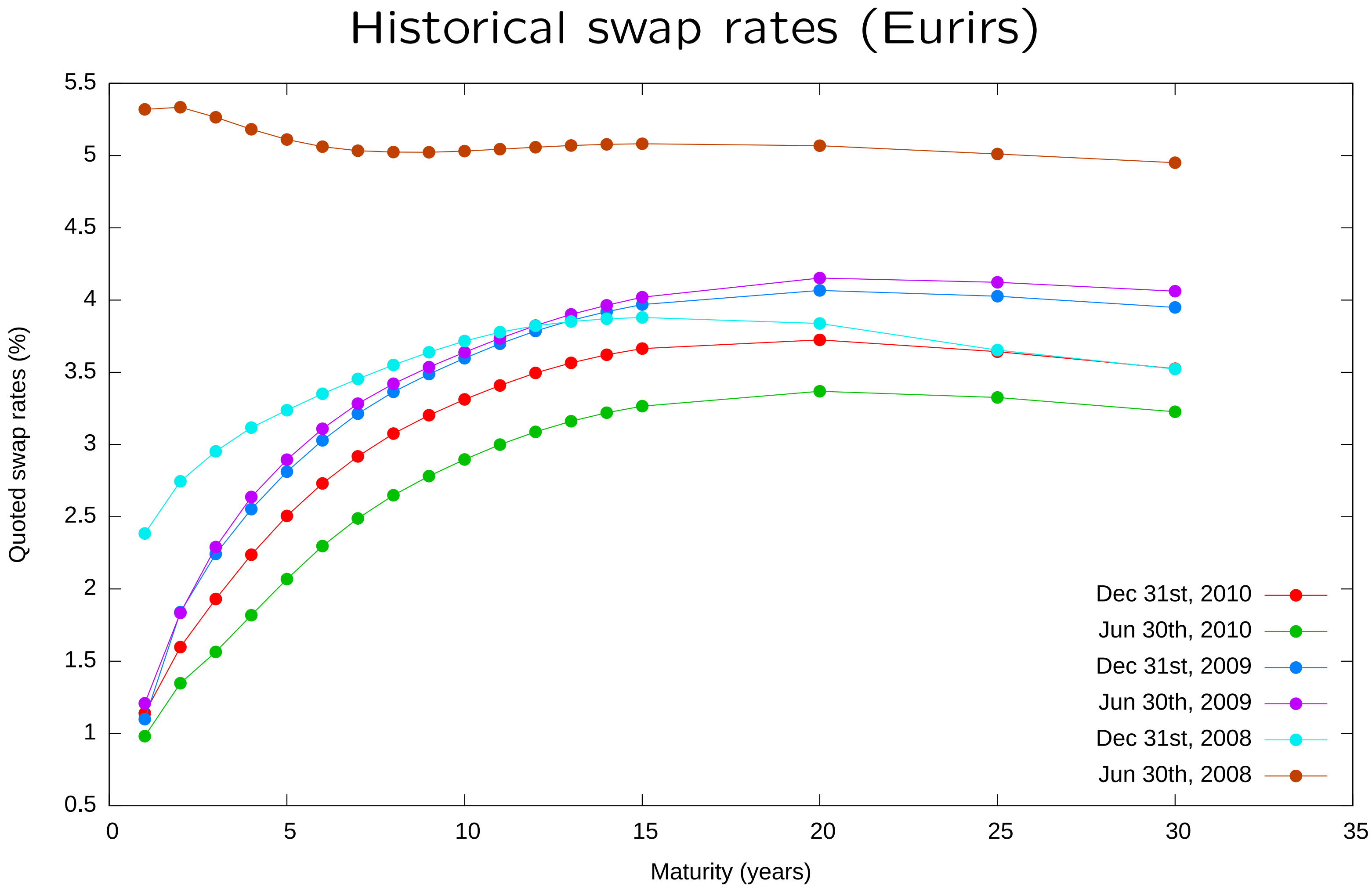
2. Creation of a spreaded curve from any yield curve

3. Computation of price for a fixed-rate coupon bond

4. Examples using the QuantLib Addin and QuantLibXL

Interest rate curve

- Building block of financial engineering
- Used to compute the present value of cash flows paid at future known dates
- Should be calibrated from market quotes
- We assume the risk-free curve to be calibrated using swap rates


Historical swap rates (Eurirs)
Quoted swap rates (%)
Maturity (years)
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
5
10
15
20
25
30
35
Dec 31st, 2010
Jun 30th, 2010
Dec 31st, 2009
Jun 30th, 2009
Dec 31st, 2008
Jun 30th, 2008

Bootstrap of a yield curve: many choices

- Choose the financial instruments that will make the curve up: quoted fair swap rates (otherwise deposit rates, IR futures, ...)

- Choose a financial quantity to interpolate: discount rate (otherwise zero rate, forward rate, ...)

- Choose an interpolator: log-linear (otherwise linear, forward flat, backward flat, ...)

Bootstrap of a swap curve in the QuantLib Addin

Create swap-rate helpers for the bootstrap

```
qlSwapRateHelper2("swap-10y",
    Rate, Tenor, Calendar, FixedLegFrequency,
    FixedLegConvention, FixedLegDayCounter, IborIndex,
    Spread, ForwardStart, DiscountingCurve) -> "swap-10y#0000"
```

Create and bootstrap the curve

```
qlPiecewiseYieldCurve("swap-YTS", NDays,
           Calendar, RateHelpers, DayCounter,
           Jumps, JumpDates, Accuracy, TraitsID,
           InterpolatorID) -> "swap-YTS#0000"
```

Discount curve as a spreaded swap curve

Given the yield curve we can compute the *"risk-free"* discount at a future date T

$$D_{\mathcal{S}}(T) = D(T; S_{1Y}, \ldots, S_{30Y})$$

The *"risky"* discount can be computed applying an extra credit spread z to the swap curve

$$D^z_{\mathcal{S}}(T) = e^{-z\,T} D_{\mathcal{S}}(T)$$

Spreaded discount curve using the QuantLib addin

Computing the discount factor using the QuantLib addin

```
qlYieldTSDiscount("swap-YTS", DfDates,
                  AllowExtrapolation) -> 0.981536
```

Using the QuantLib addin to create a spreaded discount curve

```
qlForwardSpreadedTermStructure("discount-YTS"
                               "swap-YTS", zspread)
    -> "discount-YTS#0000"
```


Credit risk modeled with credit spread

Market participants assign a penalty to credit risk.
We assume that

- Quoted financial instruments are at equilibrium for credit risk
- For each bond a single credit spread represents the risk-neutral expectation of credit risk
- Prices are obtained discounting cash flows on spreaded risk-free yield curves

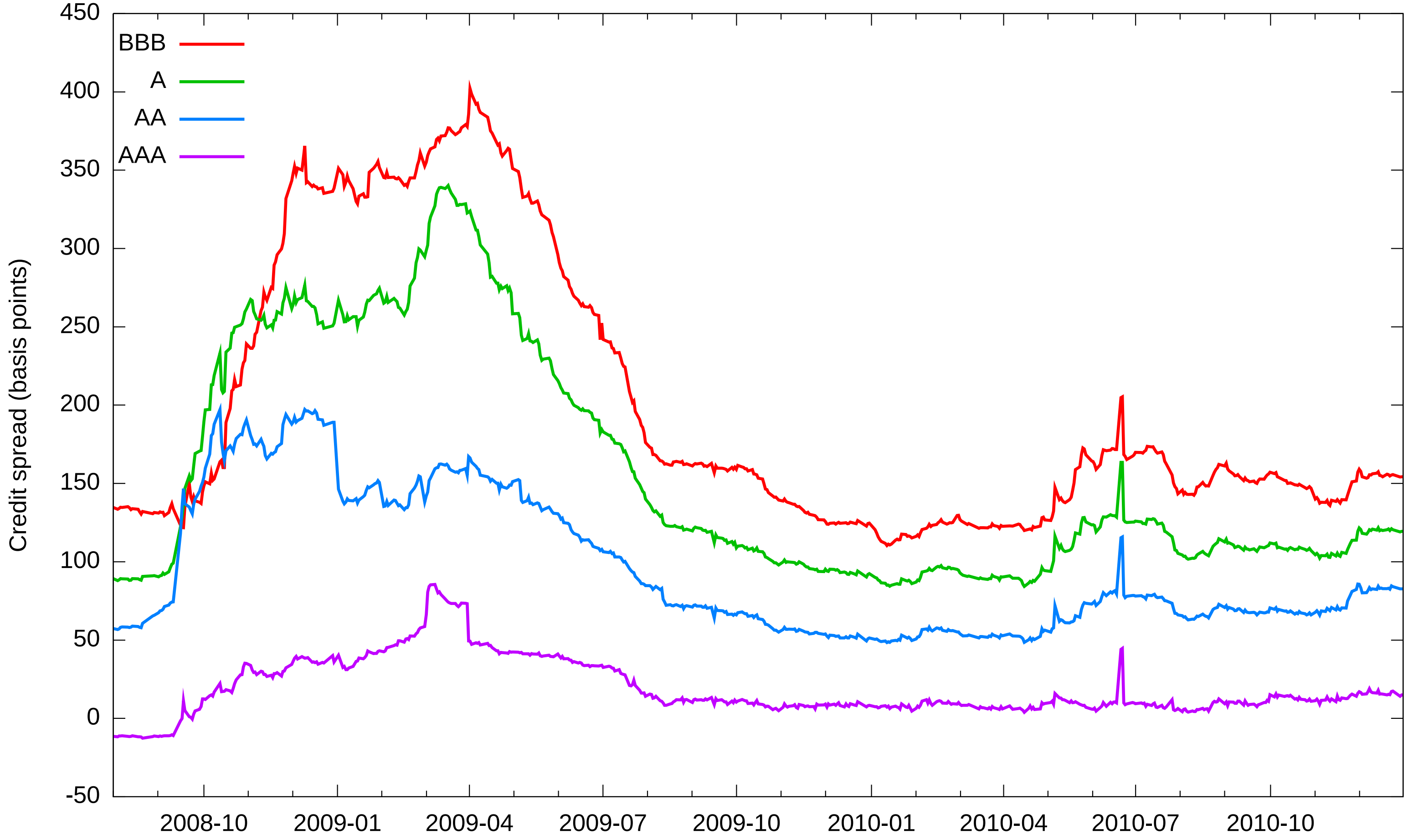
Pricing fixed-rate coupon bond

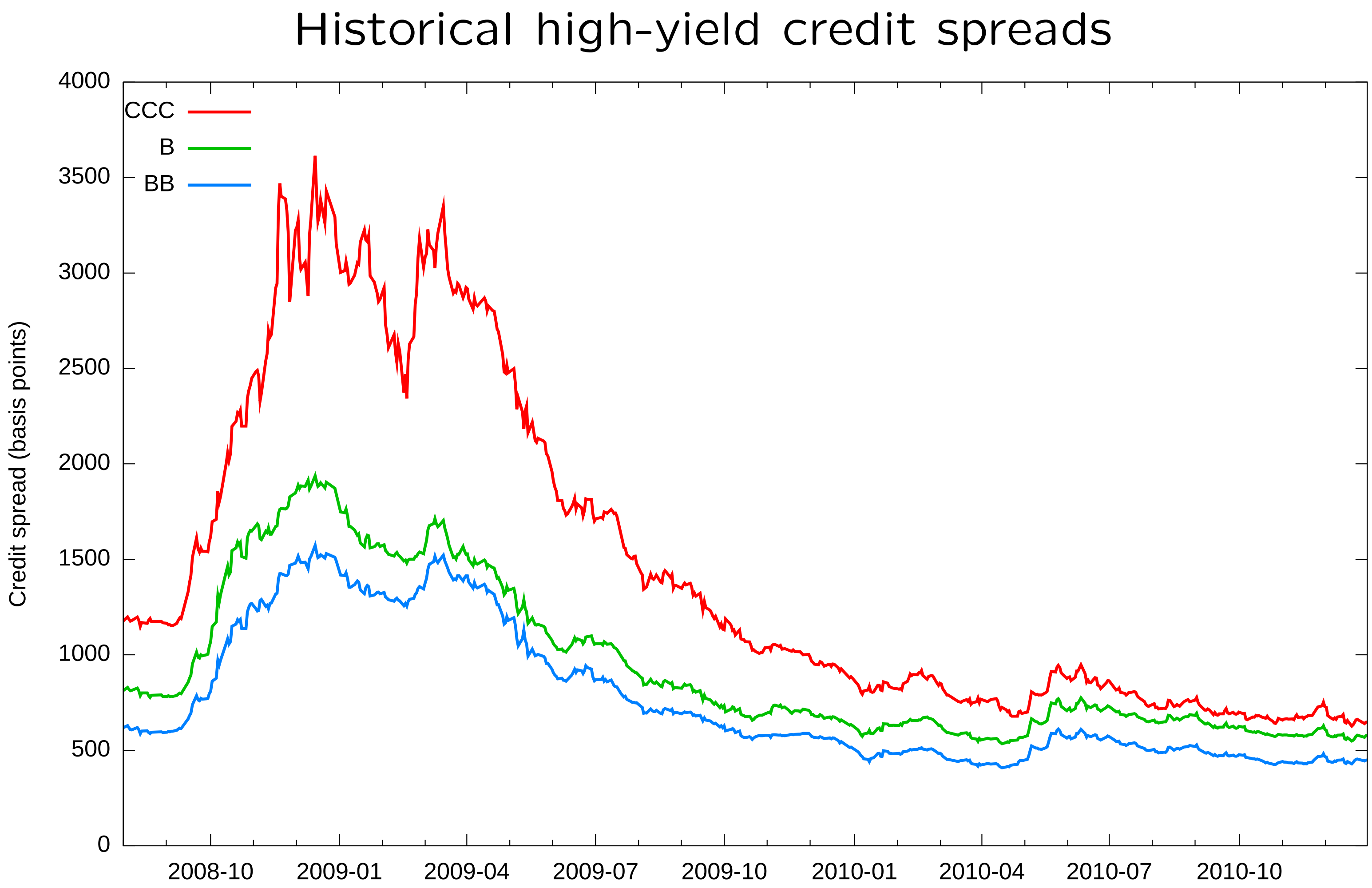
The price of a fixed-rate coupon bond can be computed discounting future coupons on a risky discount curve

$$\begin{aligned} P(\mathcal{S}, z) &= C_1\, e^{-z\, T_1} D_{\mathcal{S}}(T_1) + \ldots + [C_n + R]\; e^{-z\, T_n} D_{\mathcal{S}}(T_n) \\ &= C_1\, D^z_{\mathcal{S}}(T_1) + \ldots + [C_n + R]\; D^z_{\mathcal{S}}(T_n) \end{aligned}$$

- Assume the credit spread z to be what is needed to match quoted bond prices
- The credit spread can be observed for bonds quoted for issuers of different sectors, currencies, and ratings

Historical investment-grade credit spreads




Historical high-yield credit spreads
Credit spread (basis points)
0
500
1000
1500
2000
2500
3000
3500
4000
CCC
B
BB
2008-10
2009-01
2009-04
2009-07
2009-10
2010-01
2010-04
2010-07
2010-10

Pricing fixed-rate coupon bond in QuantLib addin

Create the fixed-rate coupon bond and its engine

```
bond = qlFixedRateBond("bond", Description, Currency,
            SettlementDays, FaceAmount, ScheduleID,
            Coupons, DayCounter, PaymentBDC,
            Redemption, IssueDate, PaymentCalendar, ...)

engine = qlBondEngine("bond-engine", "discount-curve")
```

Set the engine and compute the price

```
qlInstrumentSetPricingEngine("bond", "bond-engine")

clean-price = qlBondCleanPrice("bond")
```


Part II: Risk simulations

1. Simulations of swap curves

2. Static simulations of credit spreads

3. Dynamic simulations of credit spreads

4. Simulations of bond prices

5. Computation of risk figures

Risk-management simulations

Given a bond we would like to come up with realistic scenarios for its price at a future date (e.g. tomorrow)

- We consider the financial factors affecting the price: the risk factors
- We create simulation scenarios for each risk factor and compute the bond price assuming the realization of each scenario
- The array of computed prices can be used to compute risk-management quantities such as *value at risk*

Historical simulations of swap curves

We do not know what the swap rates will be tomorrow. However, based on the recent past, e.g. two years, we can simulate a number of possible scenarios for tomorrow's swap rates.
Assumptions:

- Tomorrow's swap rates will be similar to today's rates
- The (absolute) variations of swap rates observed historically from day to day are applied to current swap rates to create the scenarios
- In each scenario assume the swap rates as simulated and bootstrap the interest-rate curve to obtain a discount curve

Historical scenarios for swap rates

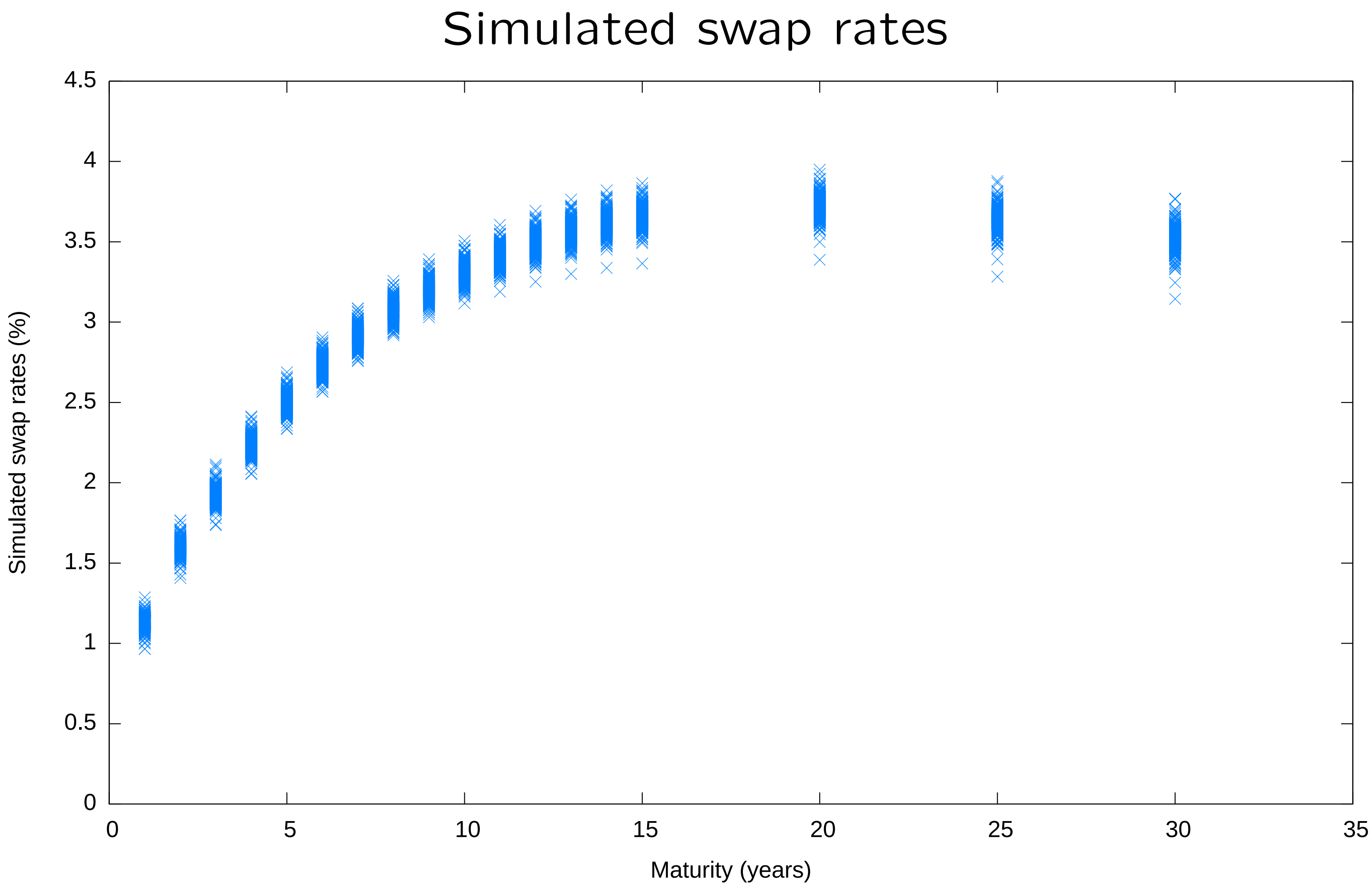
For each historical date i consider the increment in the swap rates with respect to the previous day

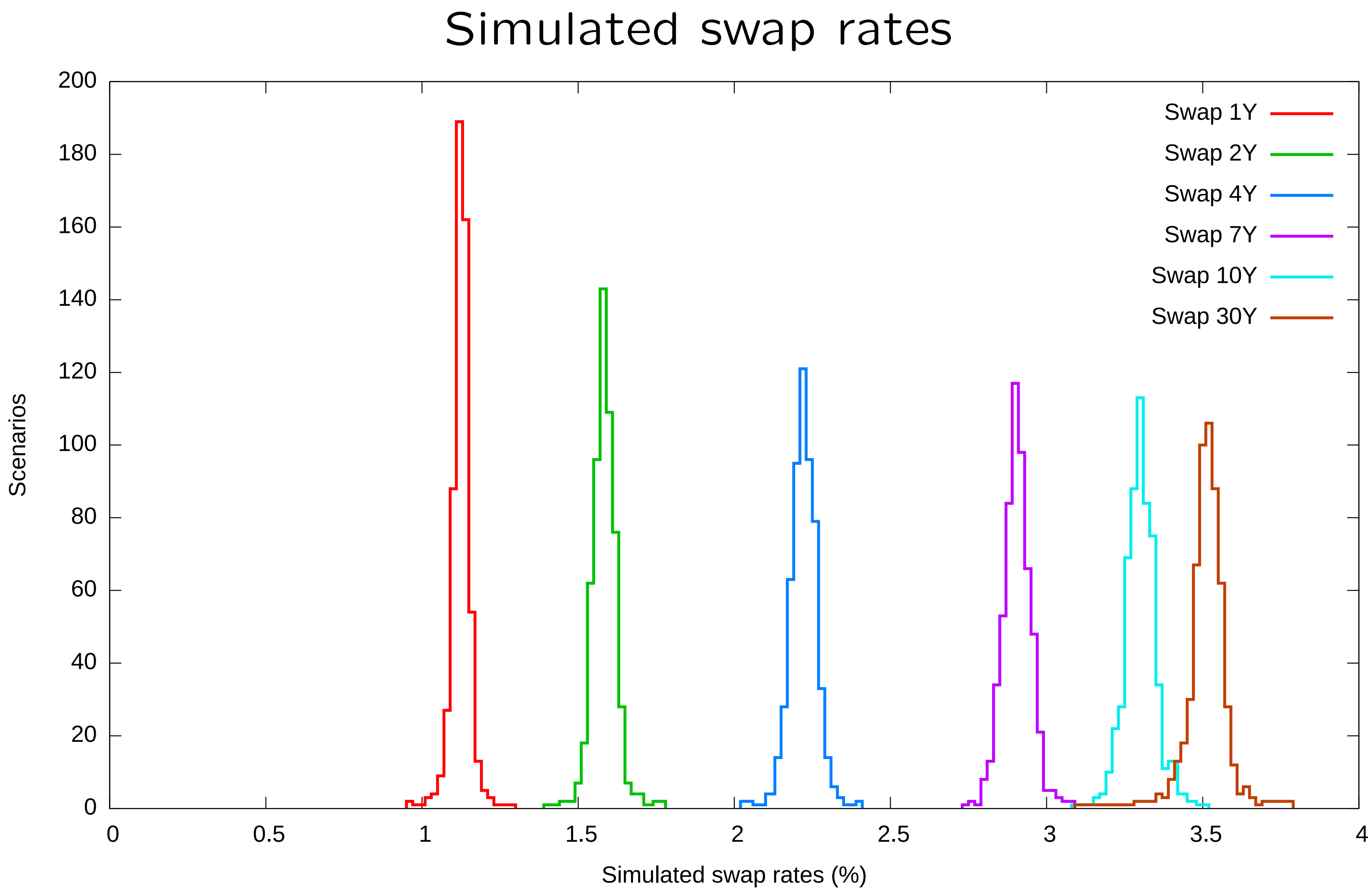
$$\Delta S^i_{1Y} = S^i_{1Y} - S^{i-1}_{1Y} \qquad \ldots \qquad \Delta S^i_{30Y} = S^i_{30Y} - S^{i-1}_{30Y}$$

To compute the swap scenarios, apply these increments to the current rates (i=0)

$$S^i_{\text{sim}(1Y)} = S^0_{1Y} + \Delta S^i_{1Y} \qquad \ldots \qquad S^i_{\text{sim}(30Y)} = S^0_{30Y} + \Delta S^i_{30Y}$$

Note: none of the swap scenarios was ever observed on the market


Simulated swap rates
Simulated swap rates (%)
Maturity (years)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
5
10
15
20
25
30
35


Simulated swap rates
Scenarios
Simulated swap rates (%)
Swap 1Y
Swap 2Y
Swap 4Y
Swap 7Y
Swap 10Y
Swap 30Y
0
20
40
60
80
100
120
140
160
180
200
0
0.5
1
1.5
2
2.5
3
3.5
4

Static simulation of credit spreads

1. Collect credit-spread indices on issuers with the same rating, currency, and sector

2. For each bond consider the corresponding index according to currency/sector and issuer rating according to the agencies

3. Apply the historical difference between the index spreads at two dates to the current credit spread

Spread indices

Sectors	Sub sectors		
all sectors	all sub sectors		
financial	all financial		
	insurance	finance	bank
industrial	all industrial		
	consumer products	health care	media
	gaming and lodging	manufacturing	energy
	basic industries	technology	retail
	transportation	telecoms	property
public	all public		
	agency	provincial	supranational
	sovereign	state guaranteed	
utility	all utility		
	electric	reg transp	gas pipelines

Spread scenarios

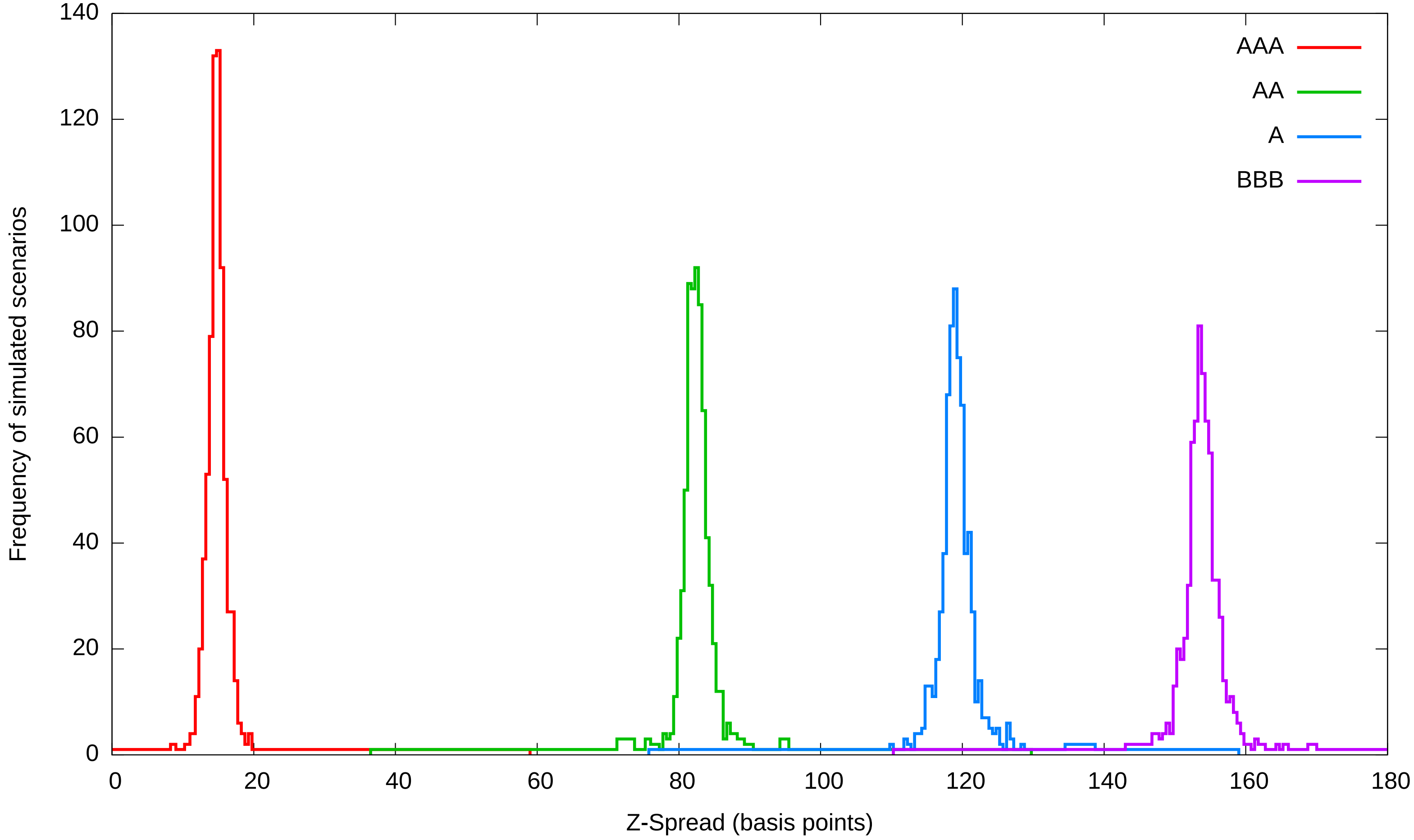
Given a maturity, a sector, a rating, and a currency from the historical spread indexes we can compute the daily variations

$$\Delta z^i_{AAA} = z^i_{AAA} - z^{i-1}_{AAA}\,; \quad \Delta z^i_{AA} = z^i_{AA} - z^{i-1}_{AA}\,; \quad \ldots$$

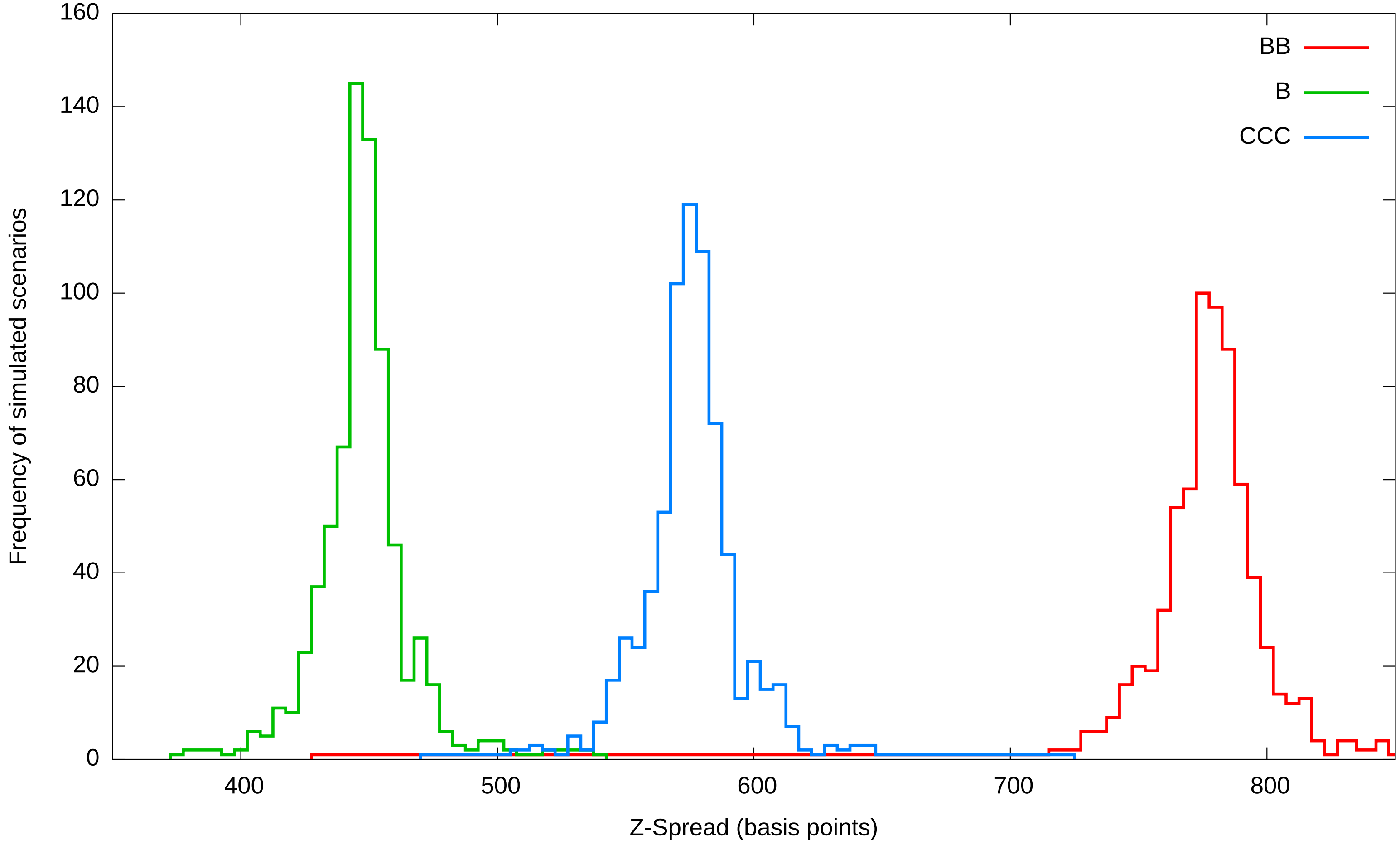
The spread scenarios are then obtained applying these variations to the current bond spread

$$z^i_{\mathsf{sim}(AAA)} = z + \Delta z^i_{AAA}\,; \quad z^i_{\mathsf{sim}(AA)} = z + \Delta z^i_{AA}\,; \quad \ldots$$

Simulated credit spreads: investment grades



Simulated credit spreads: high yield



Static Methodology—The Good,

- As widely expected, the location of the reference spreads increases with the rating worsening

- As the rating decreases the probability distributions widens and the peaks become lower

Static Methodology—The Bad, and ...

- Idiosyncratic risk is not captured as the pricing is performed using the average spread for the sector/rating/currency

- Two or more issuers in the same rating/sector/currency group will have similar spread scenarios and therefore their bond scenarios will be highly correlated

- The spread distribution reflects the rating (changing slowly) and not the market credit expectations (changing quickly)

Dynamic simulations of credit spreads

- The observed bond spread is compare with the index spread for different ratings

- When the spread falls within two ratings we can determine a fractional rating

- Bond-spread simulations can be obtained mixing index-spread simulations for the bracketed ratings

Fractional rating

Consider the credit spread for a bond, e.g. $z = 48.9$, and compare it with the index spreads for the same sector and currency for all ratings

$$z^0(\text{AAA}) = 15.0 < 48.9 < 82.7 = z^0(\text{AA})$$

Compute the spread fractions of each bracketing spread, in the example 50% and 50%, and define the fractional rating

$$50\%\ \text{AAA} + 50\%\ \text{AA}$$

For this bond we can define the spread scenarios as

$$z^i = 50\%\ z^i_{\text{sim}(AAA)} + 50\%\ z^i_{\text{sim}(A)}$$

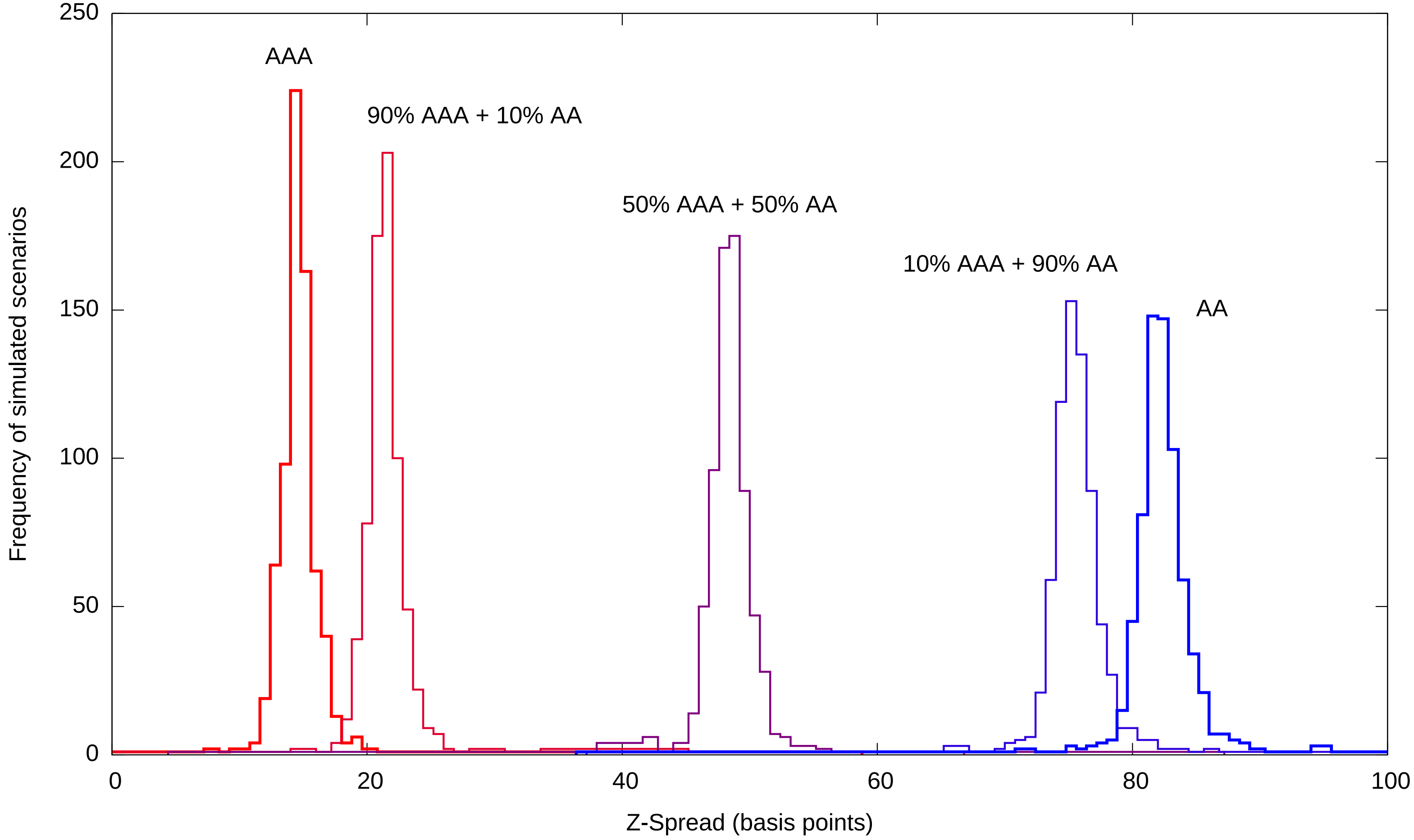
Dynamic scenarios improvements

- The response to changes in credit deterioration, or improvement, are immediately reflected in a higher, or smaller, credit rating

- From day to day, as the fractional rating changes, the spread distribution changes in shape and position

- Spread scenarios evolve and become closer to the *"correct"* scenarios **without** the explicit intervention of rating agencies

Example: three different fractional ratings



Historical simulations of bond prices

We show the StatPro version of the historical-simulation framework

- Instrument price depends deterministically from its risk factors

- In each simulation prices are *at equilibrium* and observed risk factor match simulated ones

- Risk factors are simulated starting from the current quote

- Bond prices computed using the simulated risk factors approximate the instrument distribution of future prices

Pricing function for a coupon bond

Recall that for a bond with coupons $C_1, \ldots, C_n$ the current price is given by

$$P^0 = C_1\, D^z_{\mathcal{S}^0}(T_1) + \ldots + [C_n + R]\; D^z_{\mathcal{S}^0}(T_n)$$

Define the simulated prices as

$$P^i = C_1\, D^{z^i}_{\mathcal{S}^i}(T_1) + \ldots + [C_n + R]\; D^{z^i}_{\mathcal{S}^i}(T_n)$$

$D^{z^i}_{\mathcal{S}^i}$ is the discount curve obtained from the simulated swap rates $\mathcal{S}^i_{\mathrm{sim}}$ and spreads z^i_{sim}

Computation of risk statistics

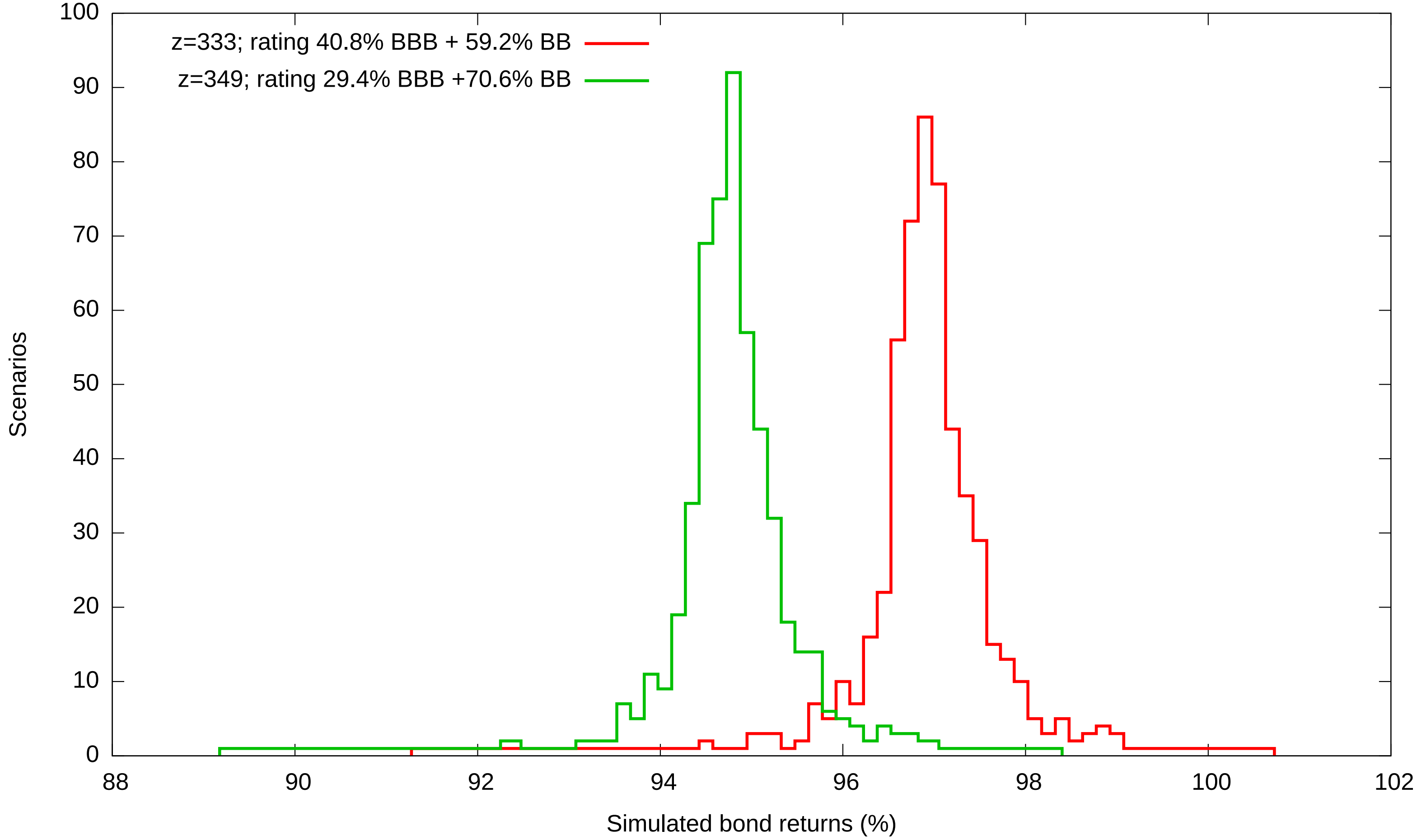
Given the current bond price P^0 and the simulated prices P^i we can consider the *distribution* of the simulated returns

$$\mathcal{D} = \left\{ \frac{P^i}{P^0} - 1 \right\}_{i=1,\ldots,N}$$

and compute the bond risk figures

$$\begin{aligned} \text{Value-at-Risk}\,(\mathcal{D}, 99\%) &= 0.63\% \\ \text{Exp-Shortfall}\,(\mathcal{D}, 99\%) &= 0.99\% \\ \text{Pot-Upside}\,(\mathcal{D}, 99\%) &= 0.81\% \end{aligned}$$

Simulated price distributions ("Fiat June 2017" and "Fiat Feb. 2021")



Risk statistics with the QuantLib addin

First create the statistic accumulator

```
stat = qlStatistics("stat", Values, Weights) -> "stat#0001"
```

Then compute the significant risk quantities

```
var95 = qlStatisticsValueAtRisk("stat", 0.95)
 es95 = qlStatisticsExpectedShortfall("stat", 0.95)
 pu95 = qlStatisticsPotentialUpside("stat", 0.95)
```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- ...

Generalization to a generic instrument

Given any (QuantLib) instrument with a pricing function f

$$P^0 = f\left(t;\, X_1, \ldots, X_K;\, r_1^0, \ldots, r_n^0\right)$$

$X_1, \ldots, X_K$ are static parameters
$r_1^0, \ldots, r_n^0$ are the observed risk factors

The simulated scenarios at time $T > t$ can be defined as

$$P^i = f\left(T;\, X_1, \ldots, X_K;\, r_1^i, \ldots, r_n^i\right)$$

$r_1^i, \ldots, r_n^i$ are the simulated risk-factor scenarios for $i = 1, \ldots, N$
N is the number of simulations

Conclusions

- We looked into the details of scenario computation for swap curve and credit spread using dynamic model
- The dynamic model has been used in StatPro since early 2007 and has performed very well during the financial crisis
- We have shown how QuantLib can be used for price computations and for risk-management simulations of a fixed-rate coupon bond

- The method shown can be generalized to the computation of risk simulations for any instrument in QuantLib (and libraries based on QuantLib, e.g. StatPro's QuantLib^2)

